Three dimensional optical transfer functions for high aperture systems with non-symmetric pupils

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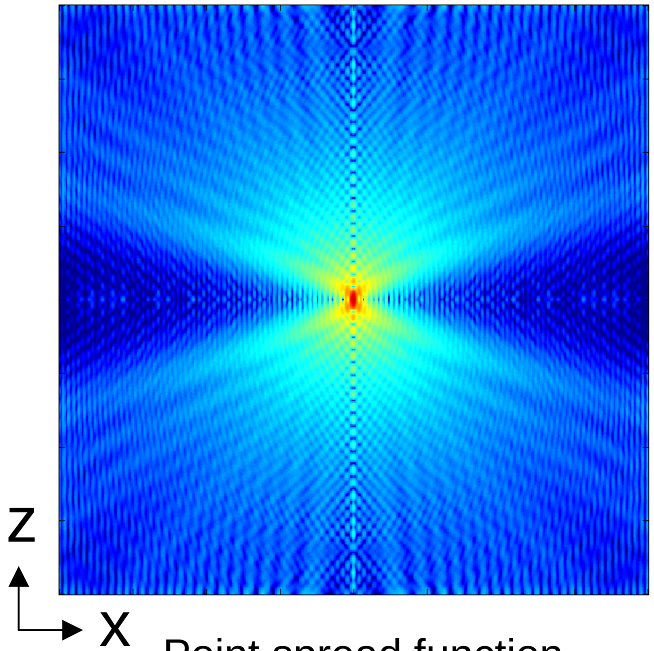
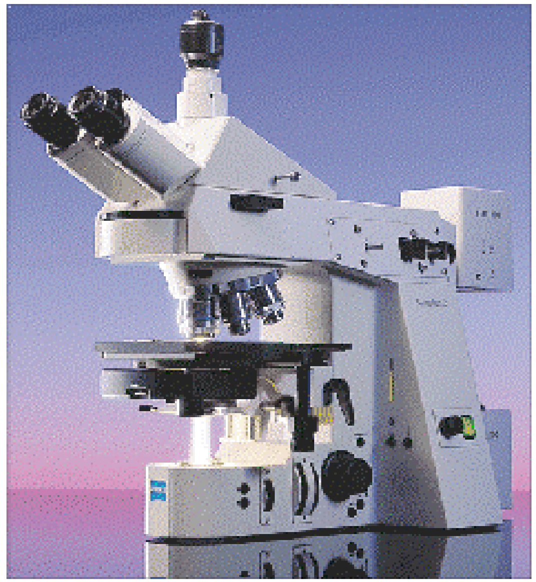
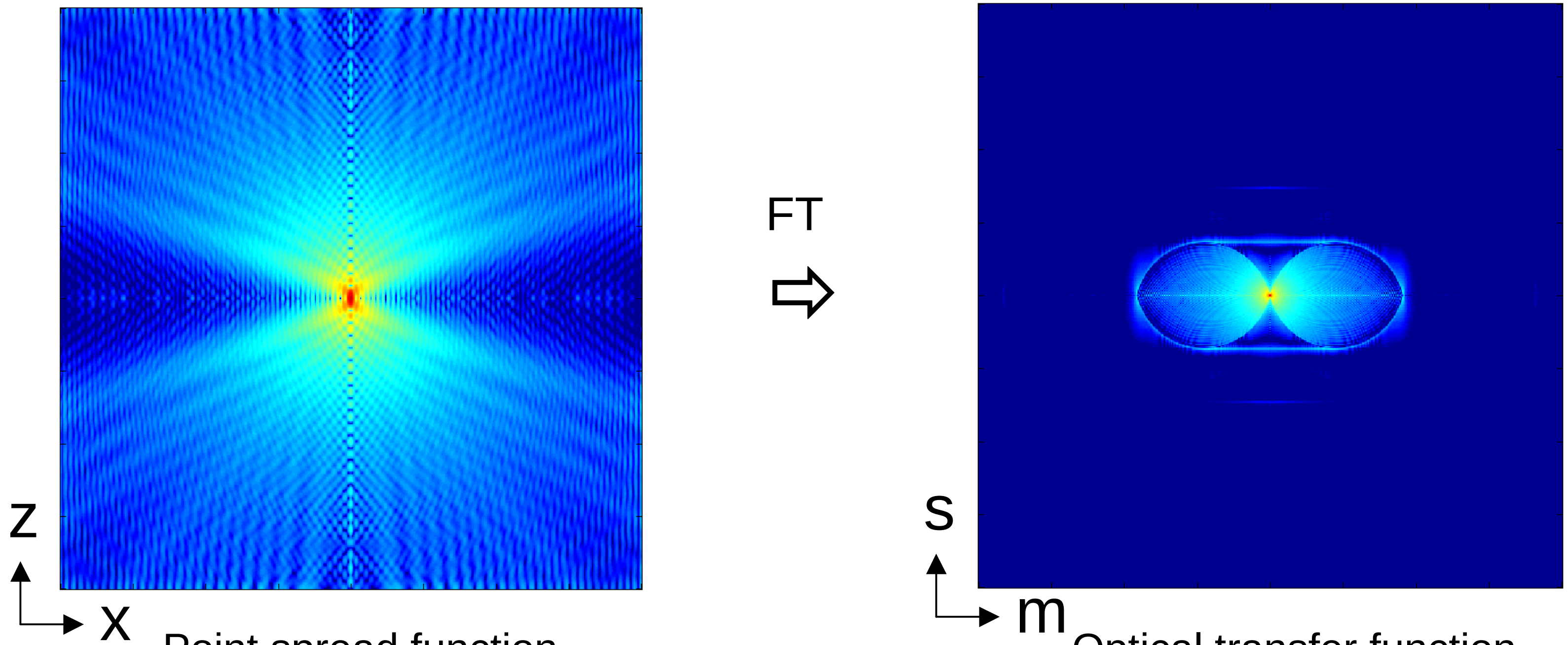
High aperture Fourier optics



Point spread function



High aperture Fourier optics



Point spread function

Optical transfer function

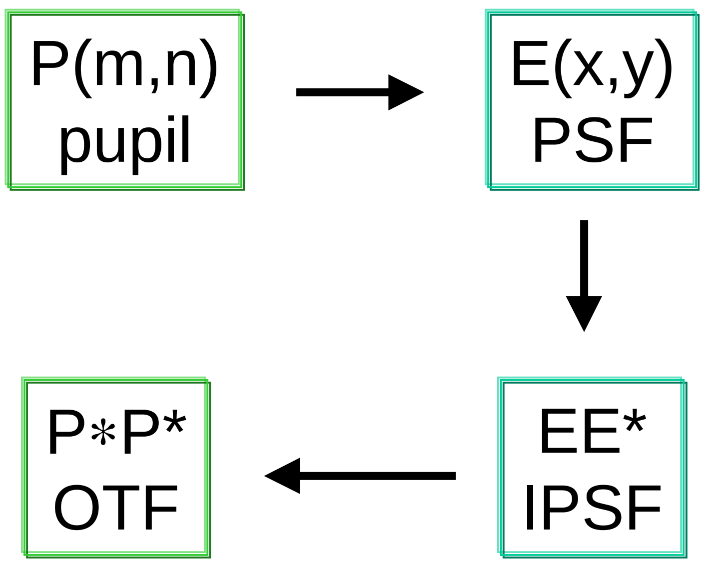
- Frieden (JOSA, **57,** p56, 1967) scalar 3D OTF assumed paraxial rays
- Sheppard (JOSA A, **11,** p593, 1994) assumed a radially symmetric pupil function
- Sheppard (Optik, **107**, p79, 1997) was vectorial but results were 2D projections

Why asymmetric?

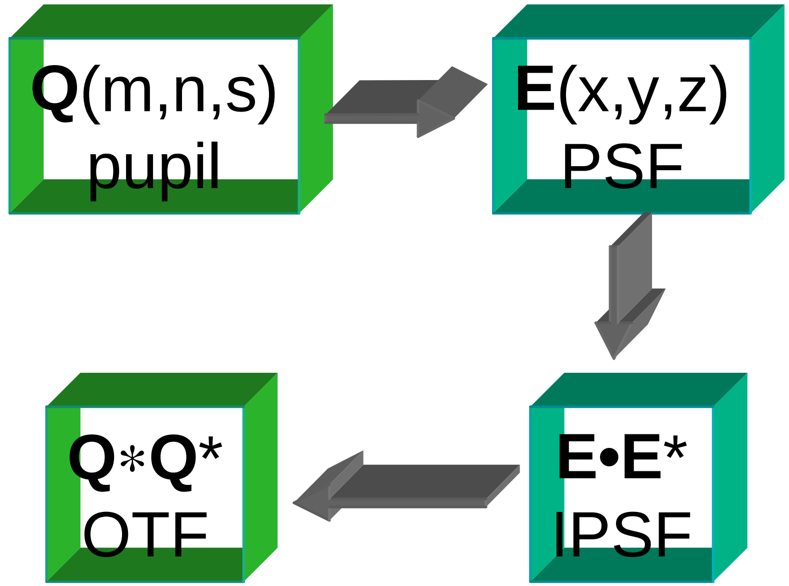
- Vectorial focussing is not radially symmetric.
- Aberrations are modeled as phase functions across the pupil which are often not radially symmetric.
- We therefore need to generalise the 3D transfer function integrals to deal with arbitrary pupil functions.

Fourier optics

2D Fourier transform



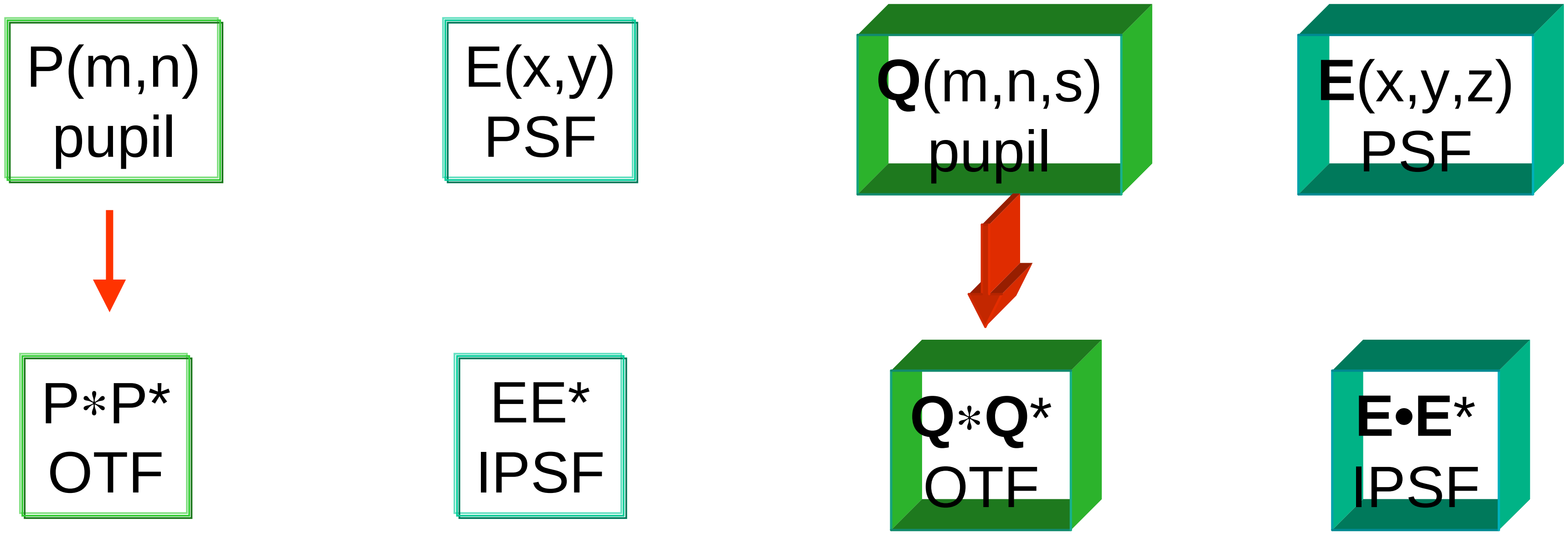
3D Fourier transform



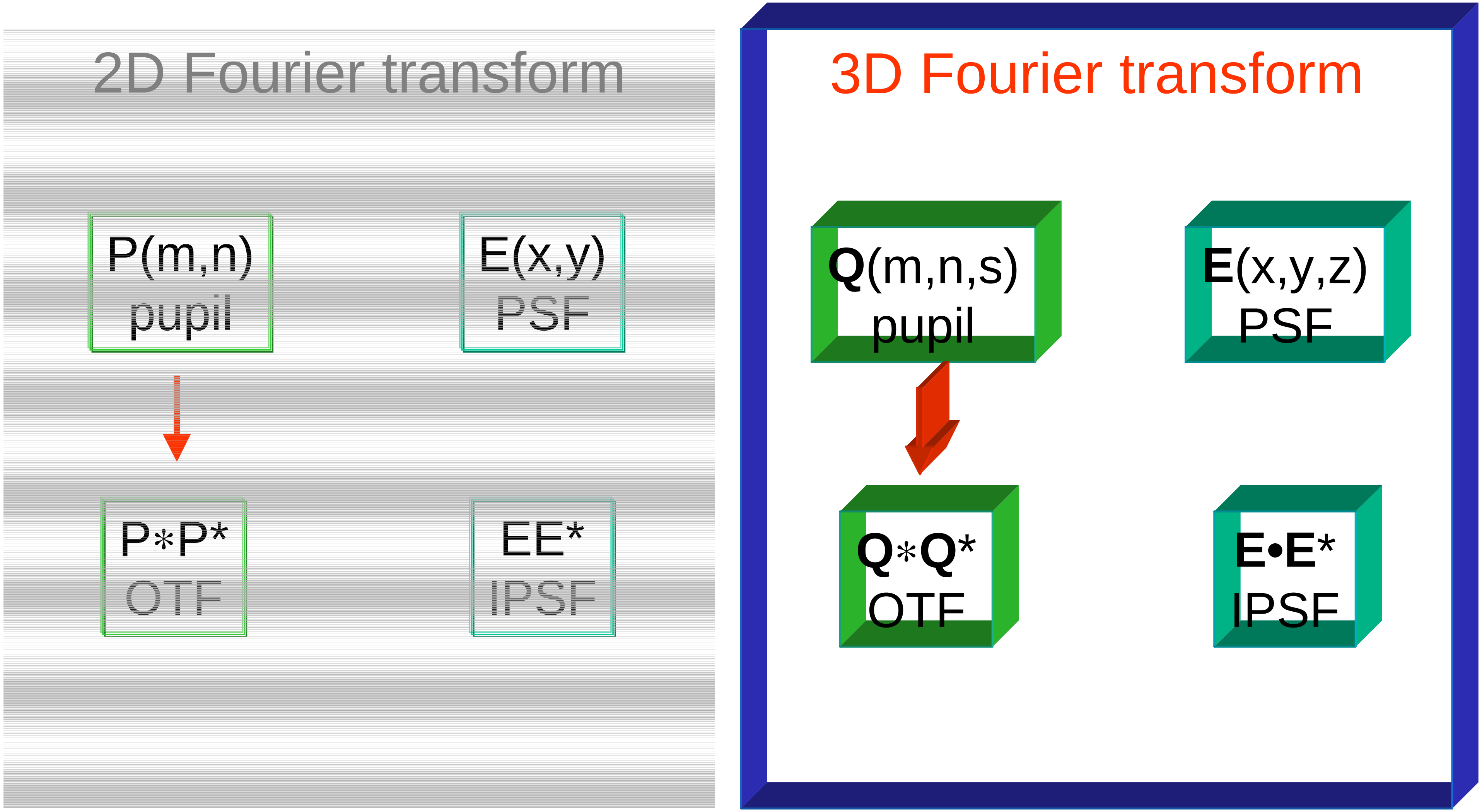
Fourier optics

2D Fourier transform

3D Fourier transform



Fourier optics



Fourier optics

Electromagnetic diffraction in optical systems

II. Structure of the image field in an aplanatic system

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The Computing Machine Laboratory, University of Manchester

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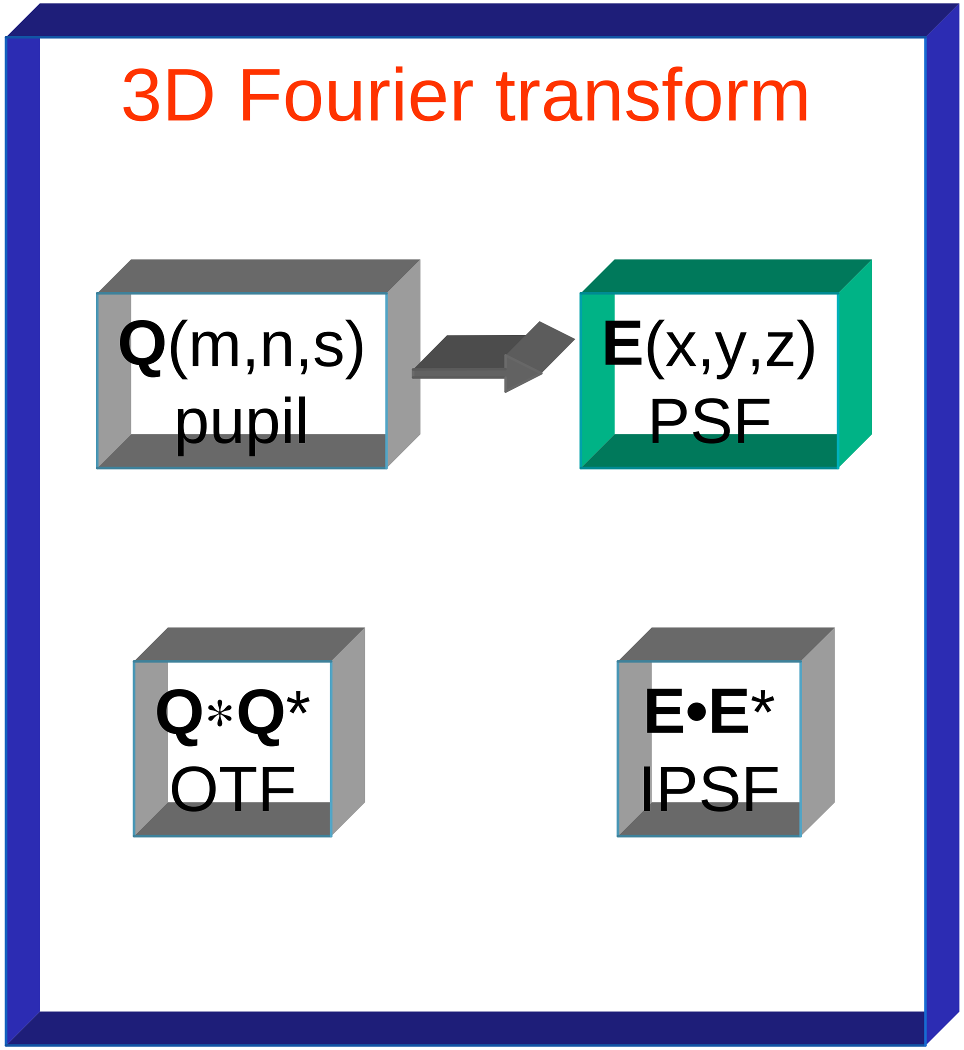
(*Communicated by D. Gabor, F.R.S.—Received* 19 *February* 1959)

An investigation is made of the structure of the electromagnetic field near the focus of an aplanatic system which images a point source. First the case of a linearly polarized incident field is examined and expressions are derived for the electric and magnetic vectors in the image space. Some general consequences of the formulae are then discussed. In particular the symmetry properties of the field with respect to the focal plane are noted and the state of polarization of the image region is investigated. The distribution of the time-averaged electric and magnetic energy densities and of the energy flow (Poynting vector) in the focal plane is studied in detail, and the results are illustrated by diagrams and in a tabulated form based on data obtained by extensive calculations on an electronic computer. The case of an unpolarized field is also investigated.

The solution is not restricted to systems of low aperture, and the computational results cover, in fact, selected values of the angular semi-aperture α on the image side, in the whole range $0 \leqslant \alpha \leqslant 90°$. The limiting case $\alpha \to 0$ is examined in detail and it is shown that the field is then completely characterized by a single, generally complex, scalar function, which turns out to be identical with that of the classical scalar theory of Airy, Lommel and Struve.

The results have an immediate bearing on the resolving power of image forming systems; they also help our understanding of the significance of the scalar diffraction theory, which is customarily employed, without a proper justification, in the analysis of images in low-aperture systems.



Fourier optics

$$e_x = -\frac{iA}{\pi}\int_0^{\alpha}\int_0^{2\pi}\cos^{\frac{1}{2}}\theta\sin\theta\{\cos\theta+(1-\cos\theta)\sin^2\phi\}\,e^{ikr_p\cos\epsilon}\,d\theta\,d\phi,$$

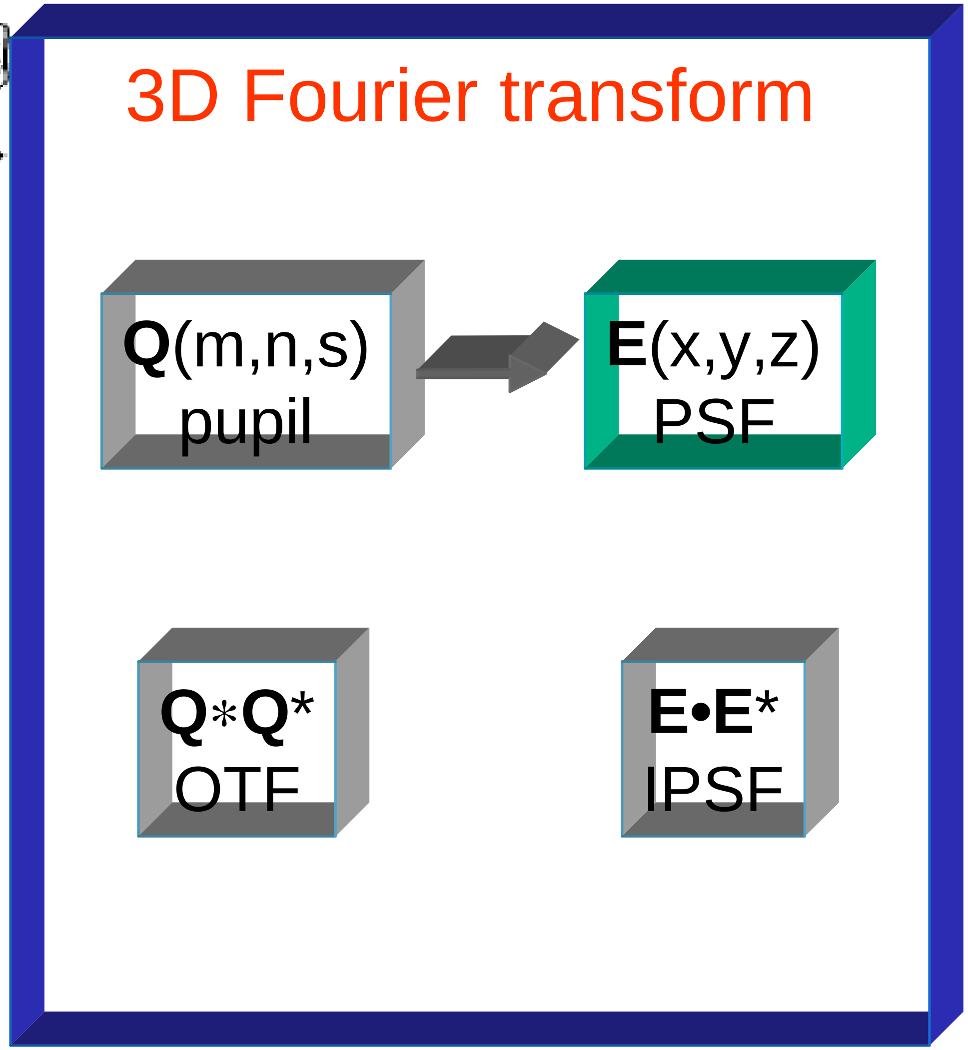
$$e_y = \frac{iA}{\pi}\int_0^{\alpha}\int_0^{2\pi}\cos^{\frac{1}{2}}\theta\sin\theta(1-\cos\theta)\cos\phi\sin\phi\,e^{ikr_p\cos\epsilon}\,d\theta\,d\phi,$$

$$e_z = \frac{iA}{\pi}\int_0^{\alpha}\int_0^{2\pi}\cos^{\frac{1}{2}}\theta\sin^2\theta\cos\phi\,e^{ikr_p\cos\epsilon}\,d\theta\,d\phi;$$

$$h_x = \frac{iA}{\pi}\int_0^{\alpha}\int_0^{2\pi}\cos^{\frac{1}{2}}\theta\sin\theta(1-\cos\theta)\cos\phi\sin\phi\,e^{ikr_p\cos\epsilon}\,d\theta\,d\phi,$$

$$h_y = -\frac{iA}{\pi}\int_0^{\alpha}\int_0^{2\pi}\cos^{\frac{1}{2}}\theta\sin\theta\{1-(1-\cos\theta)\sin^2\phi\}\,e^{ikr_p\cos\epsilon}\,d\theta\,d\phi,$$

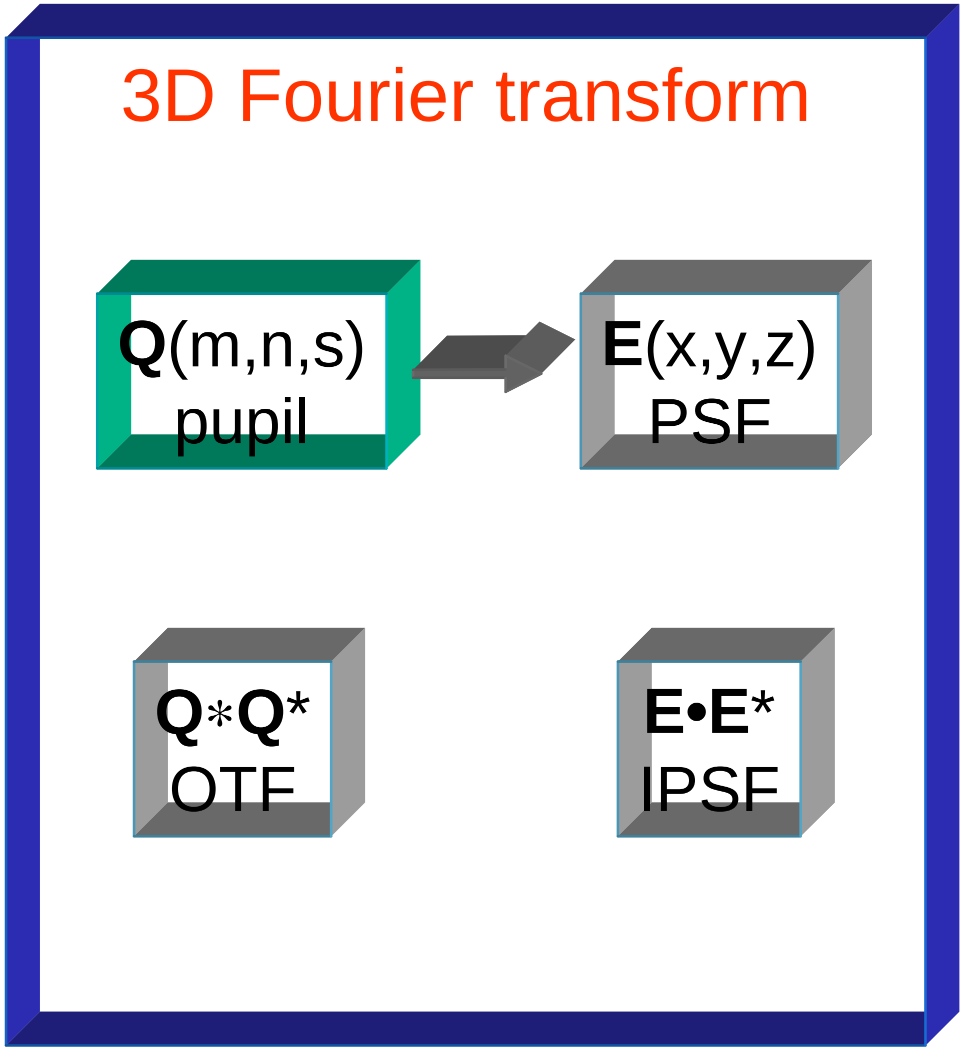
$$h_z = \frac{iA}{\pi}\int_0^{\alpha}\int_0^{2\pi}\cos^{\frac{1}{2}}\theta\sin^2\theta\sin\phi\,e^{ikr_p\cos\epsilon}\,d\theta\,d\phi.$$



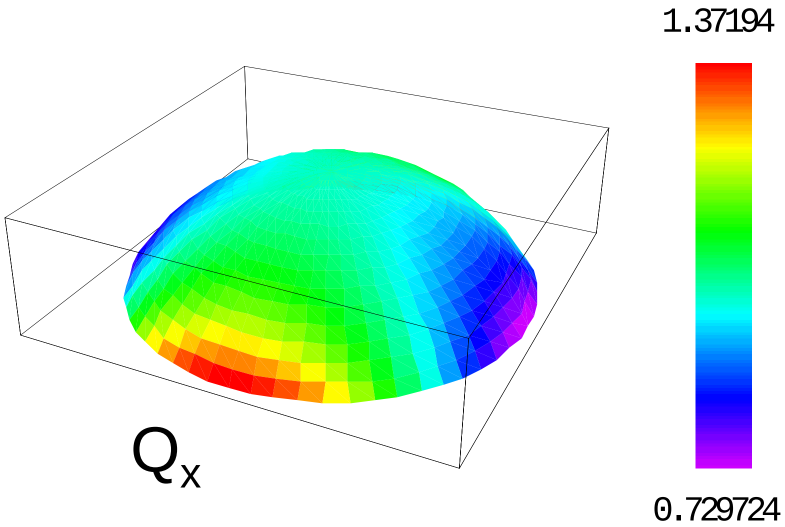
Fourier optics

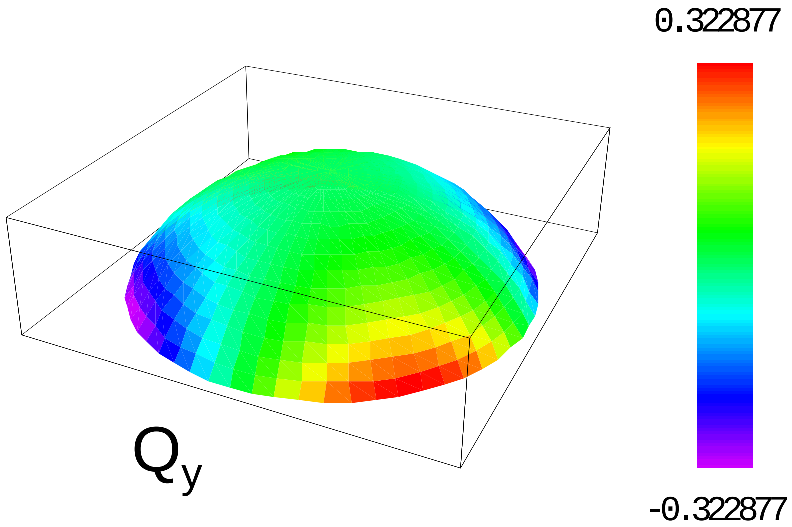
$$\left.\begin{aligned}
a_x &= f l_0 \cos^{\frac{1}{2}}\theta[\cos\theta + \sin^2\phi(1-\cos\theta)],\\
a_y &= f l_0 \cos^{\frac{1}{2}}\theta[(\cos\theta - 1)\cos\phi\sin\phi],\\
a_z &= -f l_0 \cos^{\frac{1}{2}}\theta \sin\theta\cos\phi,
\end{aligned}\right\}$$

$$\left.\begin{aligned}
b_x &= f l_0 \cos^{\frac{1}{2}}\theta[(\cos\theta - 1)\cos\phi\sin\phi],\\
b_y &= f l_0 \cos^{\frac{1}{2}}\theta[1-\sin^2\phi(1-\cos\theta)],\\
b_z &= -f l_0 \cos^{\frac{1}{2}}\theta \sin\theta\sin\phi.
\end{aligned}\right\}$$

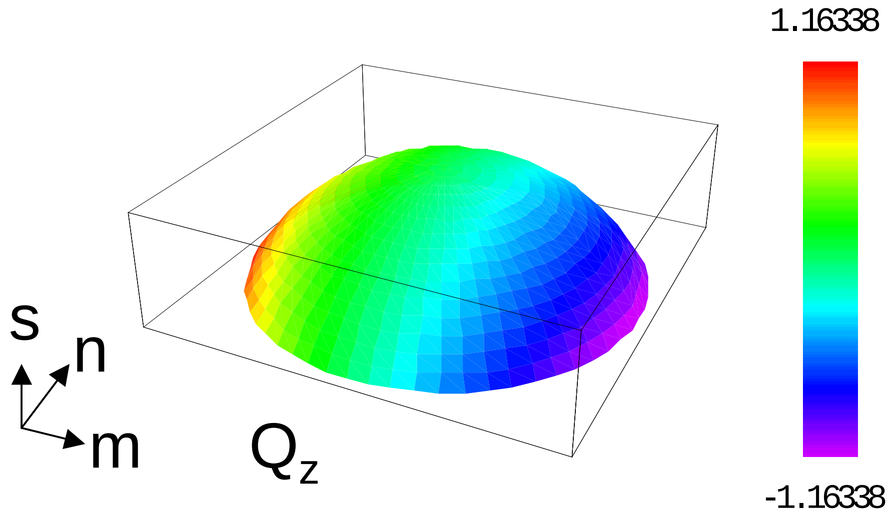


3D vectorial pupils





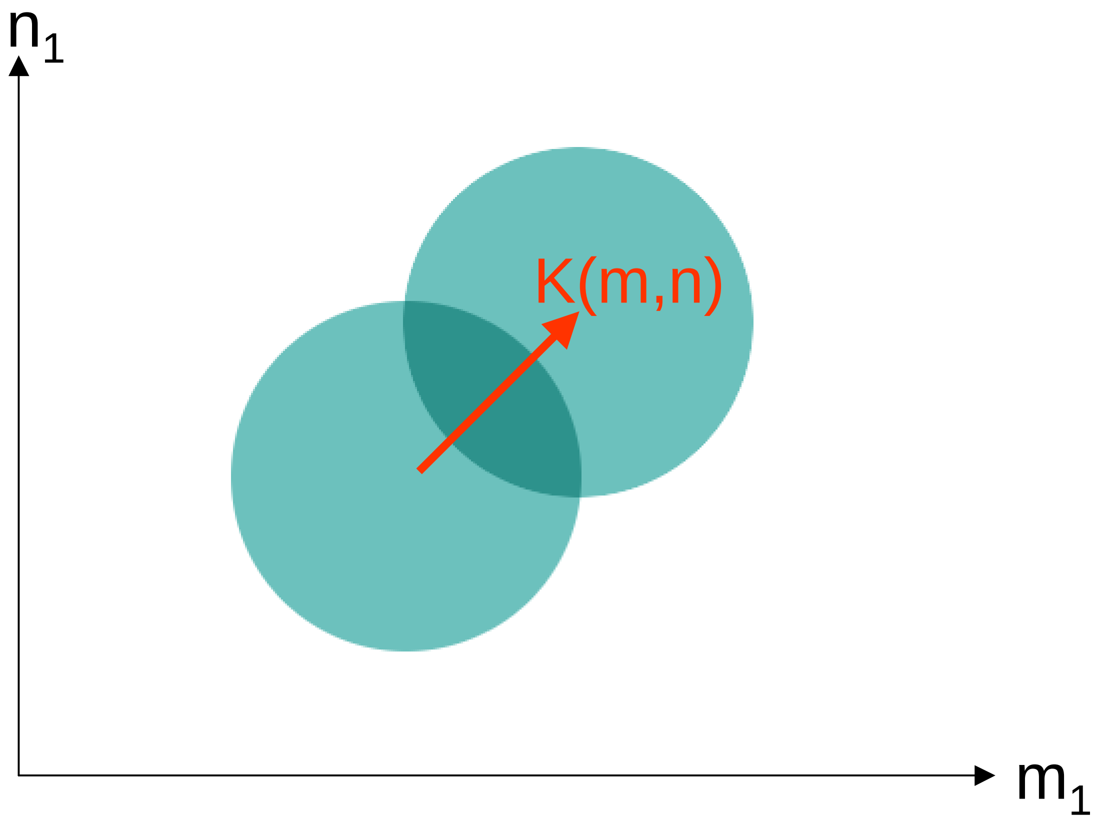
Sine apodisation
$\alpha=\pi/3$
(NA 0.87)
Input beam
x-polarised



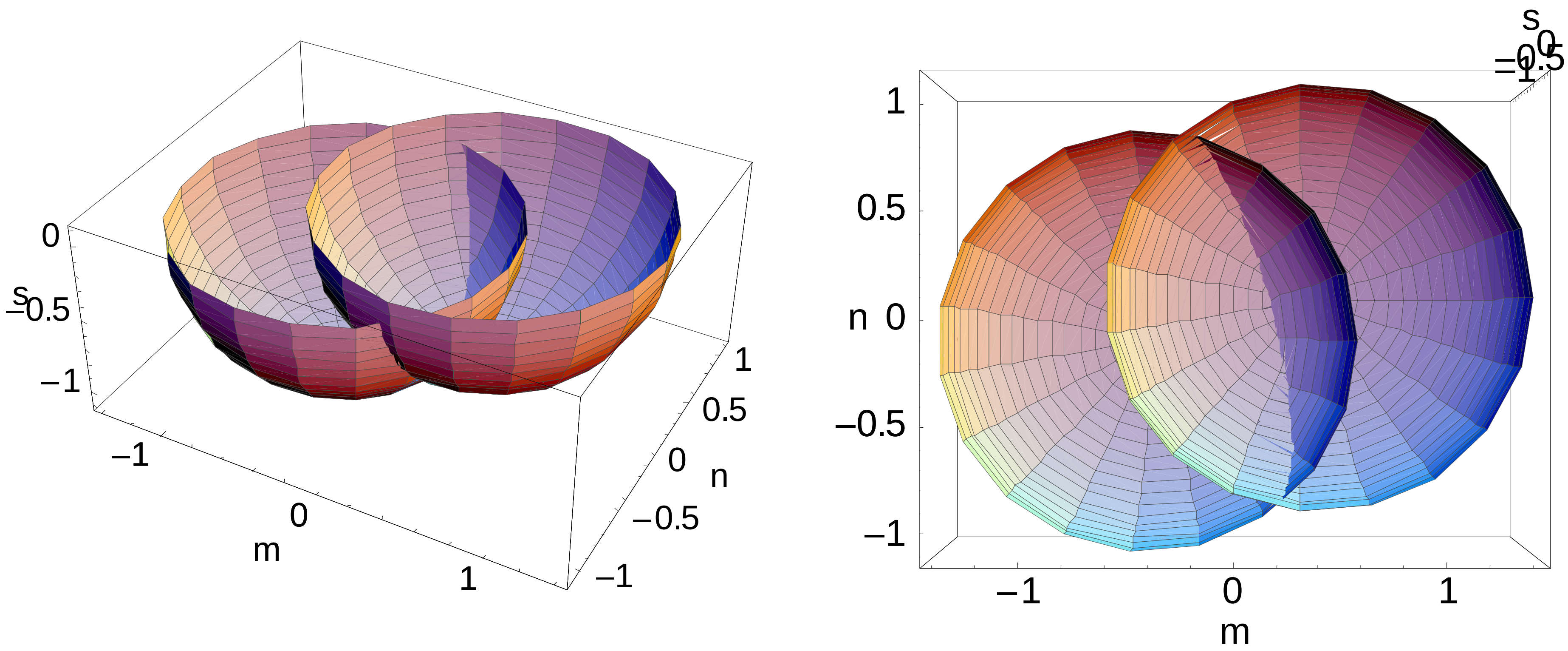
$$\vec{a}(m,n) = \begin{pmatrix} (m^2 s + n^2)/l^2 \\ -mn(1-s)/l^2 \\ -m \end{pmatrix}$$

Mansuripur *JOSA A*, 1986.
Sheppard & Larkin, *Optik*, 1997.

Correlation of 2D pupil functions

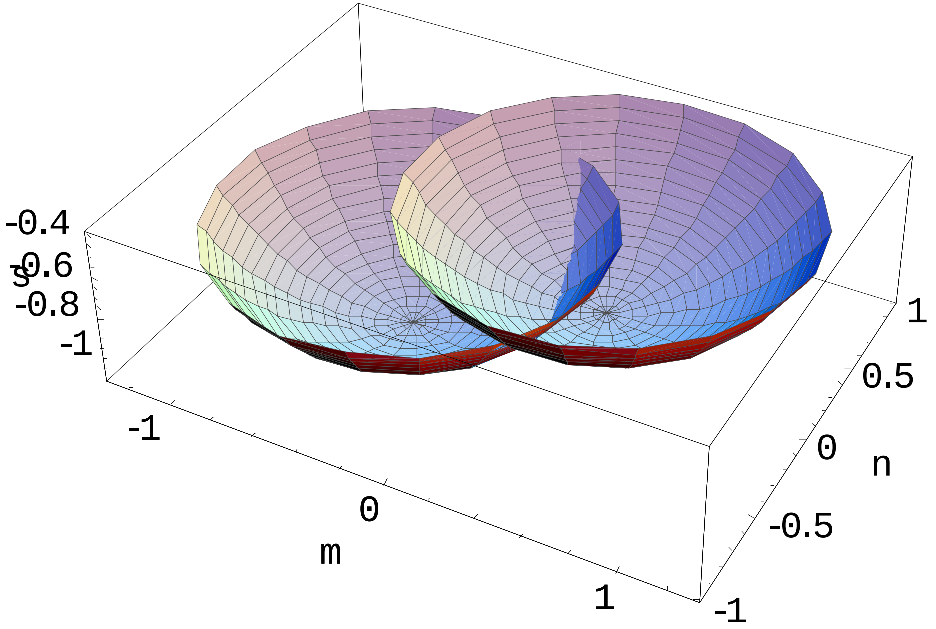


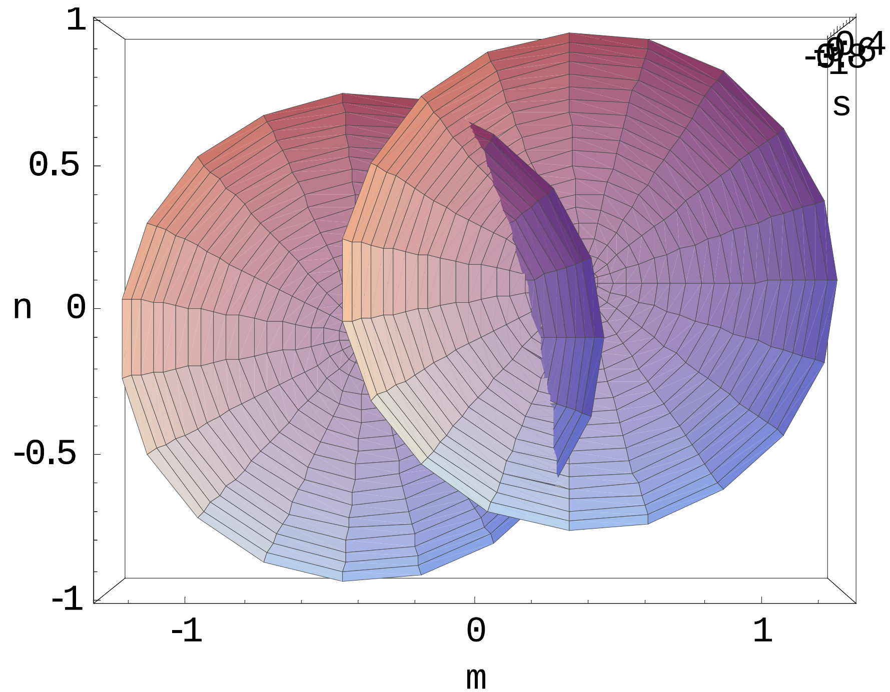
Correlation of 3D pupil functions



α=π/2

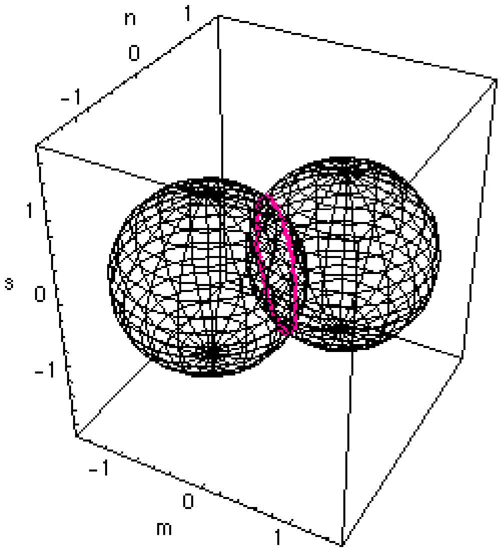
Correlation of 3D pupil functions





α=π/3

Circle of intersection



$$K = |\vec{K}| = \sqrt{m^2 + n^2 + s^2}.$$

$$l = \sqrt{m^2 + n^2}$$

$$r_0 = |\vec{r}_0| = \sqrt{1 - \frac{K^2}{4}}.$$

$$\vec{r}_0(\vec{K}, \beta) = \vec{R}_s[-\arctan(n/m)]\vec{R}_n[\pi/2 - \arccos(s/K)]\vec{r}_0(\vec{K}_0, \beta)$$

$$= \begin{pmatrix} r_0 \frac{1}{lK} [ms\cos\beta - nK\sin\beta] \\ r_0 \frac{1}{lK} [ns\cos\beta + mK\sin\beta] \\ -r_0 \frac{l}{K} \cos\beta \end{pmatrix}$$

Correlation integral

General pupil

$$\vec{Q}(\vec{m}) = \vec{P}(\vec{m})\delta(|\vec{m}| - k^2).$$

Auto-correlation

$$C(\vec{K}) = \int\int\int \vec{Q}(\vec{m} + \frac{\vec{K}}{2}) \cdot \vec{Q}^*(\vec{m} - \frac{\vec{K}}{2})\mathrm{d}\vec{m}.$$

Correlation integral

Projected pupil

$$\vec{P}_{+}(m,n) = \frac{1}{s}\vec{a}(m,n)S(m,n)T(m,n)$$

Declination Polarisation Apodisation Complex pupil mask

Auto-correlation

$$C(\vec{K}) = \frac{1}{KN(\alpha)}\int_{-\beta_1}^{\beta_1} \vec{P}(\vec{r}_0(\vec{K},\beta)+\frac{\vec{K}}{2})\cdot\vec{P}^{*}(\vec{r}_0(\vec{K},\beta)-\frac{\vec{K}}{2})\mathrm{d}\beta.$$

Normalisation

Correlation integral

Projected pupil

$$\vec{P}_{+}(m,n) = \frac{1}{s}\vec{a}(m,n)S(m,n)T(m,n)$$

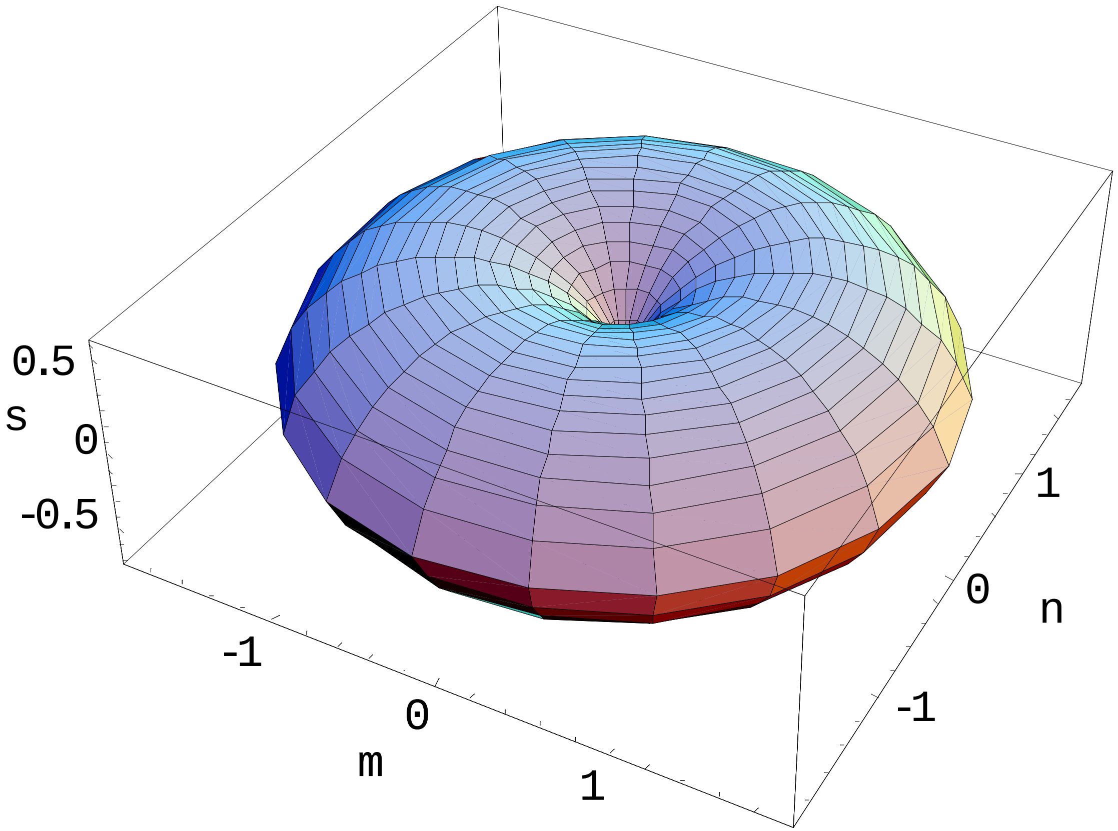
Auto-correlation

$$C(\vec{K}) = \frac{1}{KN(\alpha)}\int_{-\beta_1}^{\beta_1} \vec{P}(\vec{r}_0(\vec{K},\beta) + \frac{\vec{K}}{2}) \cdot \vec{P}^*(\vec{r}_0(\vec{K},\beta) - \frac{\vec{K}}{2})\mathrm{d}\beta.$$

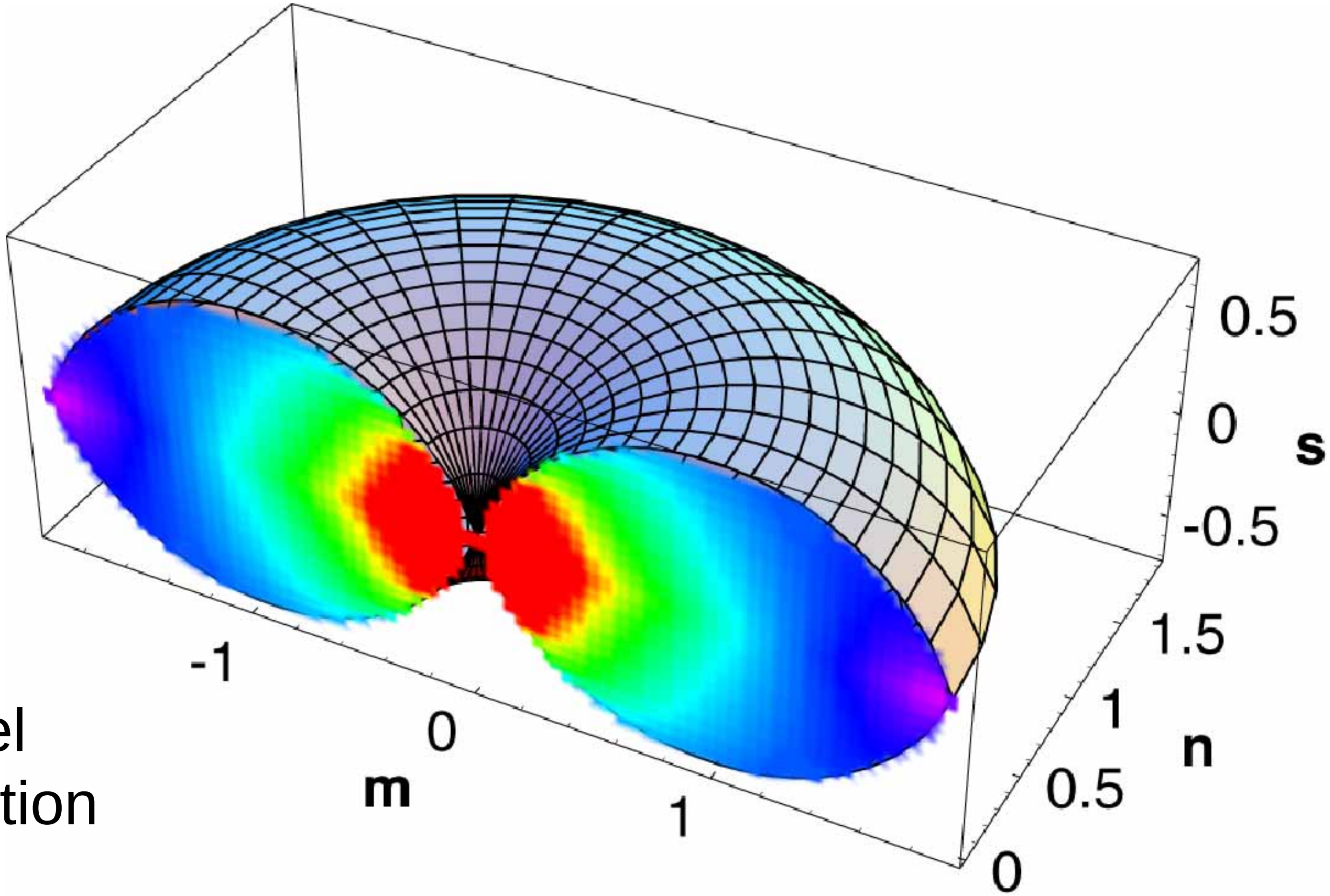
Circle of intersection

$$\vec{r}_0(\vec{K},\beta) = \vec{R}_s[-\arctan(n/m)]\vec{R}_n[\pi/2 - \arccos(s/K)]\vec{r}_0(\vec{K}_0,\beta)$$
$$= \begin{pmatrix} r_0 \frac{1}{lK}\left[ms\cos\beta - nK\sin\beta\right] \\ r_0 \frac{1}{lK}\left[ns\cos\beta + mK\sin\beta\right] \\ -r_0 \frac{l}{K}\cos\beta \end{pmatrix}$$

Widefield vectorial OTF

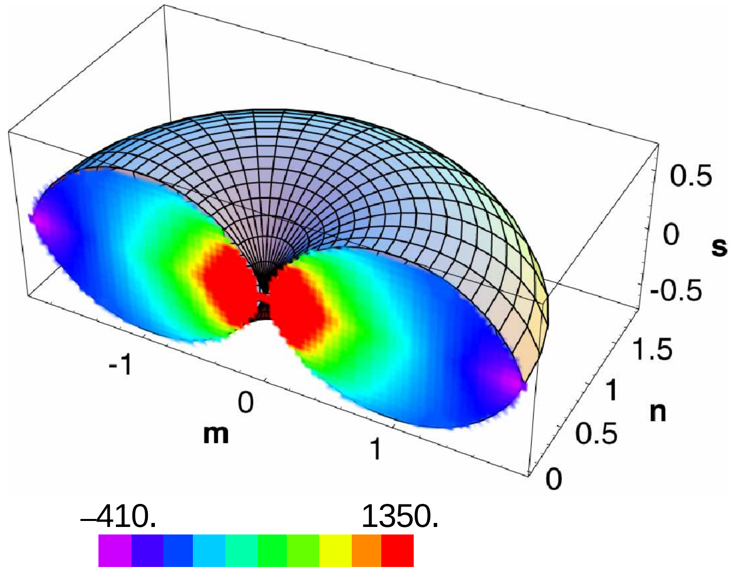


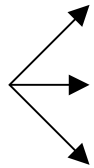
Widefield vectorial OTF

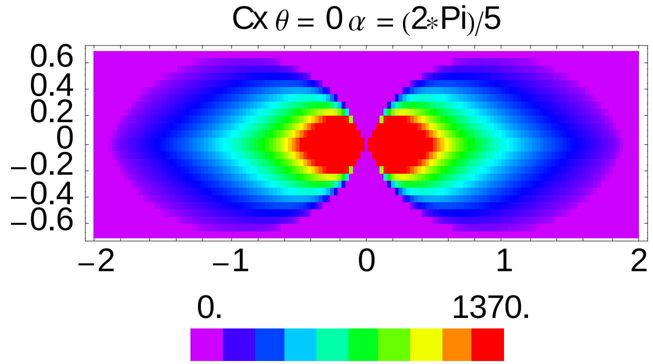


Herschel
apodisation
$\alpha=2\pi/5$
(NA 0.95)

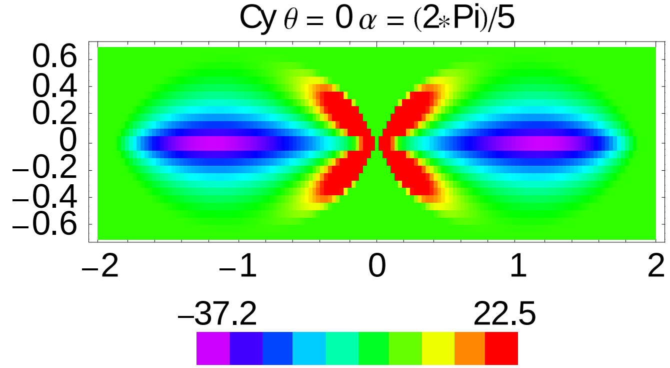
VOTF: axial slices



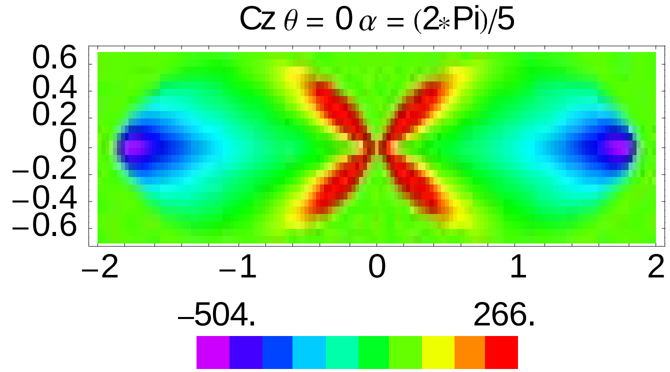




C_x

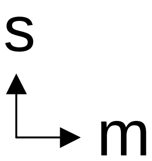


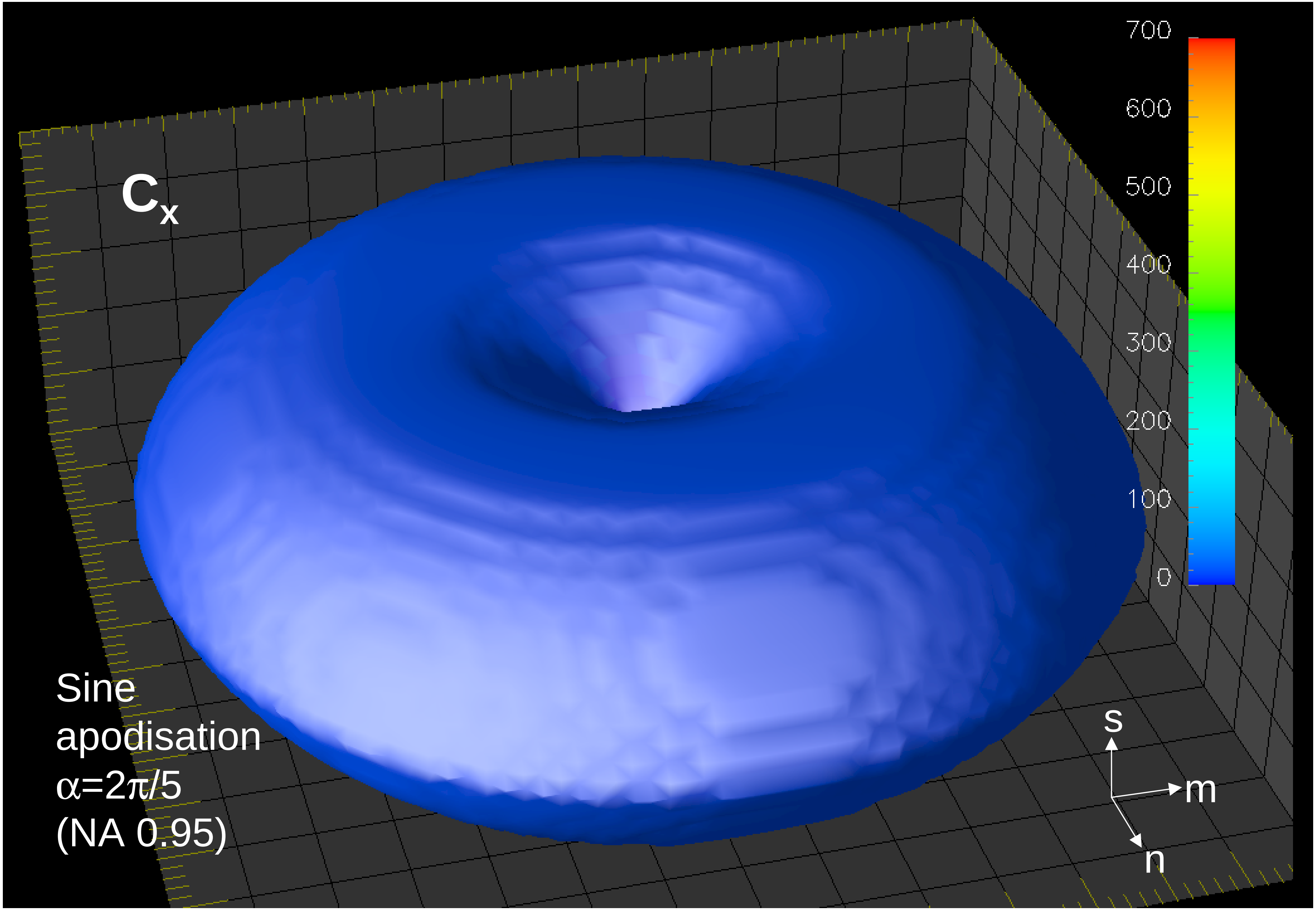
C_y



C_z

Herschel apodisation
$\alpha=2\pi/5$
n=0





Cx
700
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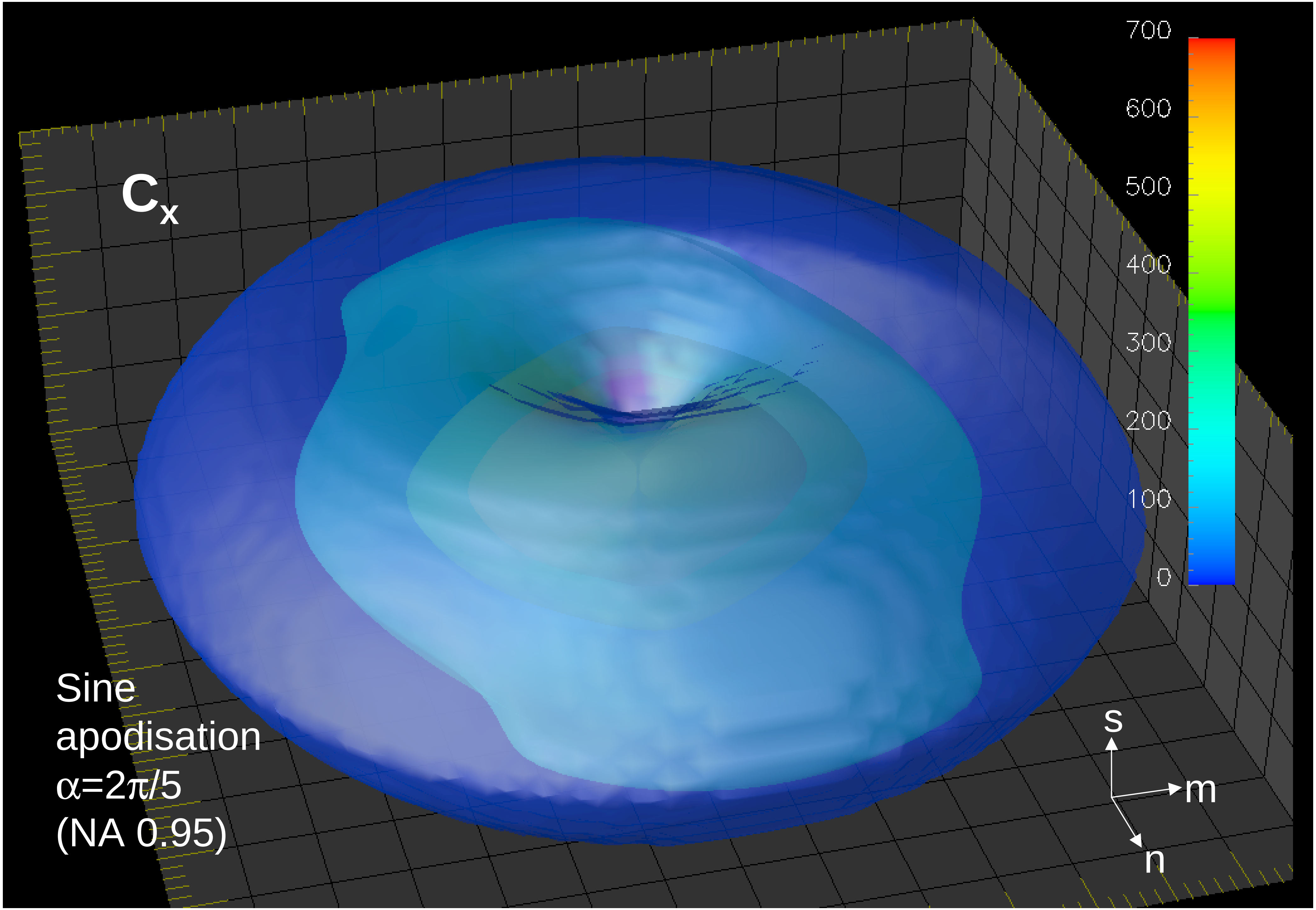
s
m
n
Sine
apodisation
α=2π/5
(NA 0.95)



C_x
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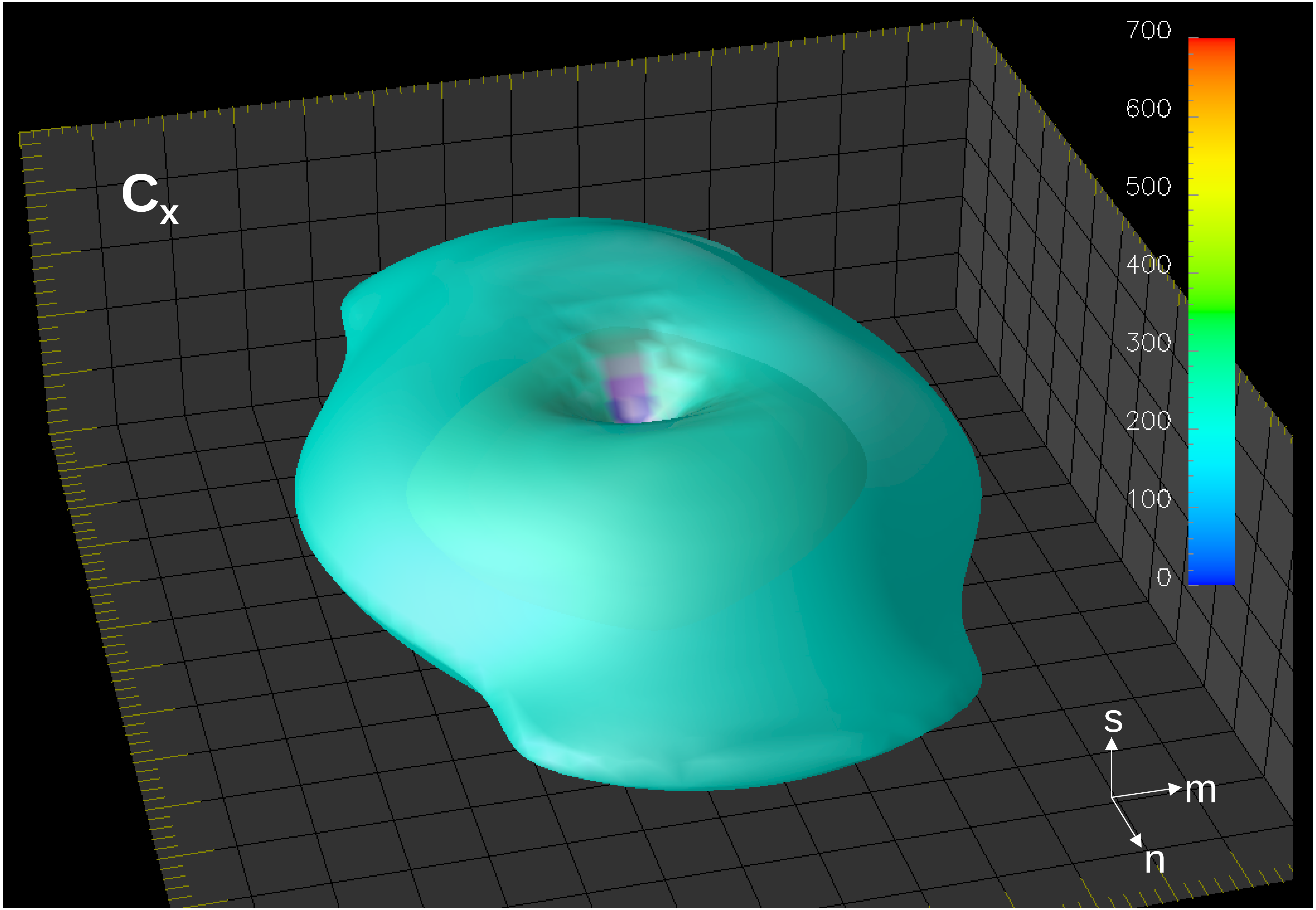
Sine
apodisation
α=2π/5
(NA 0.95)



C_x
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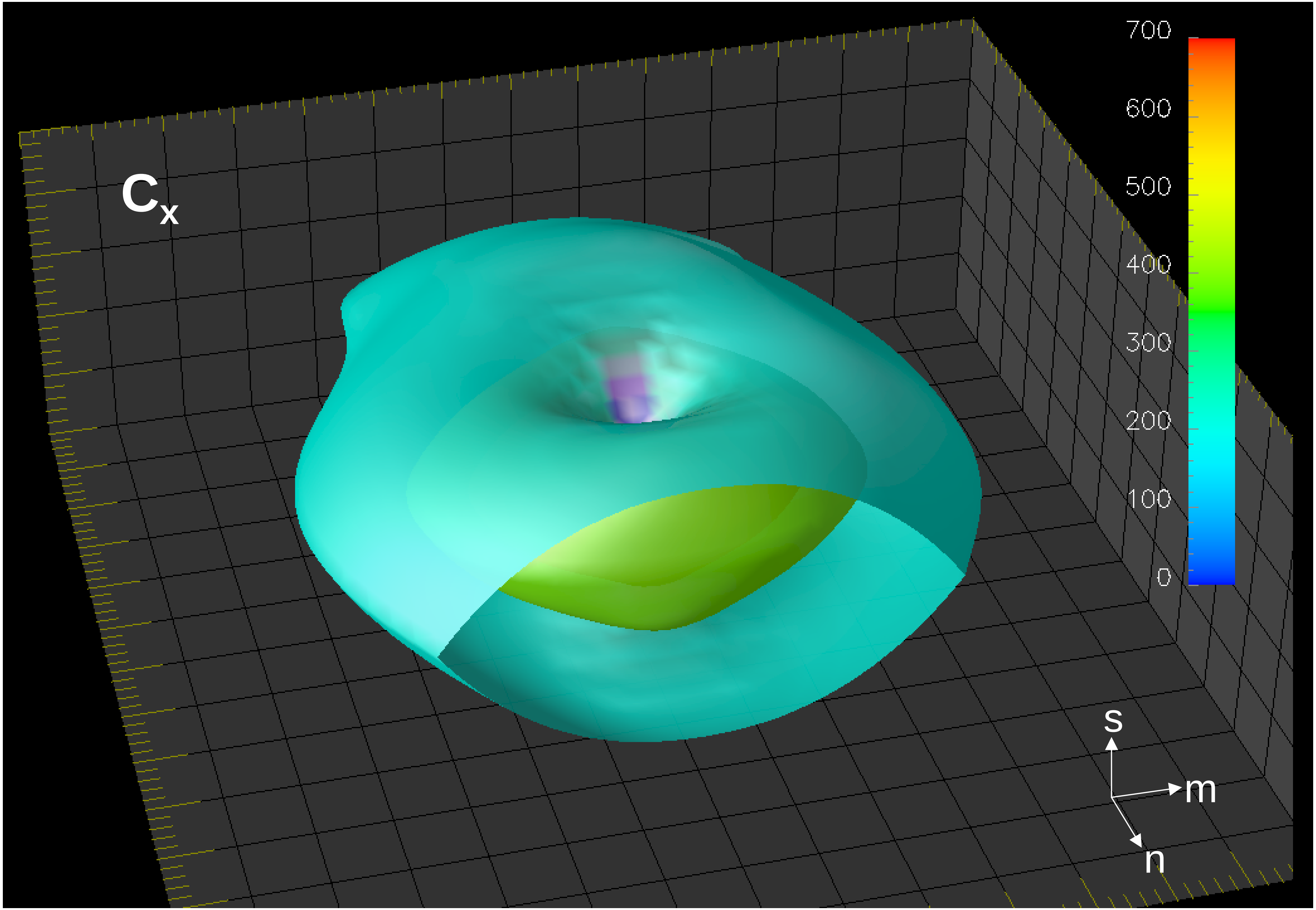

C_x
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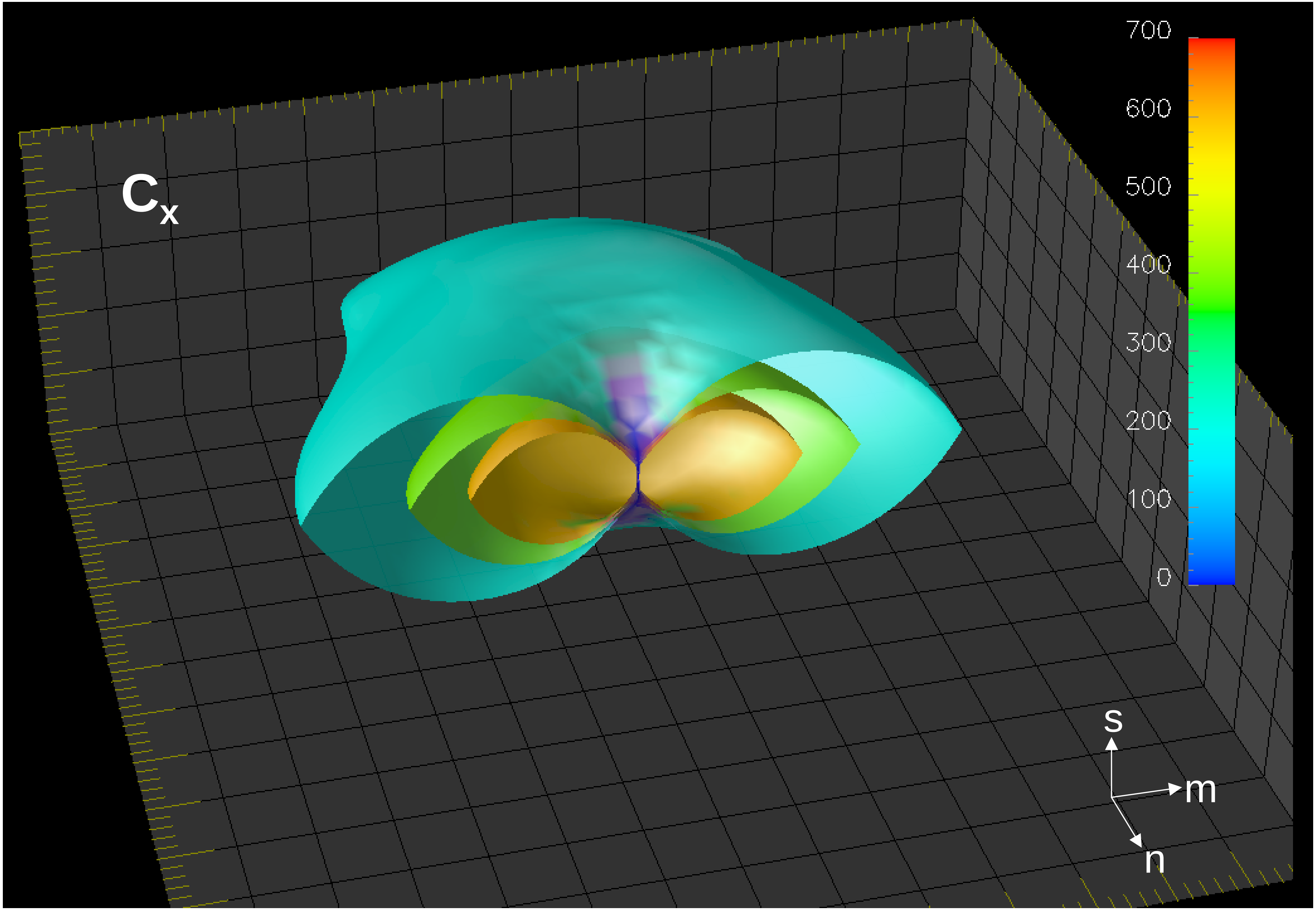

C_x
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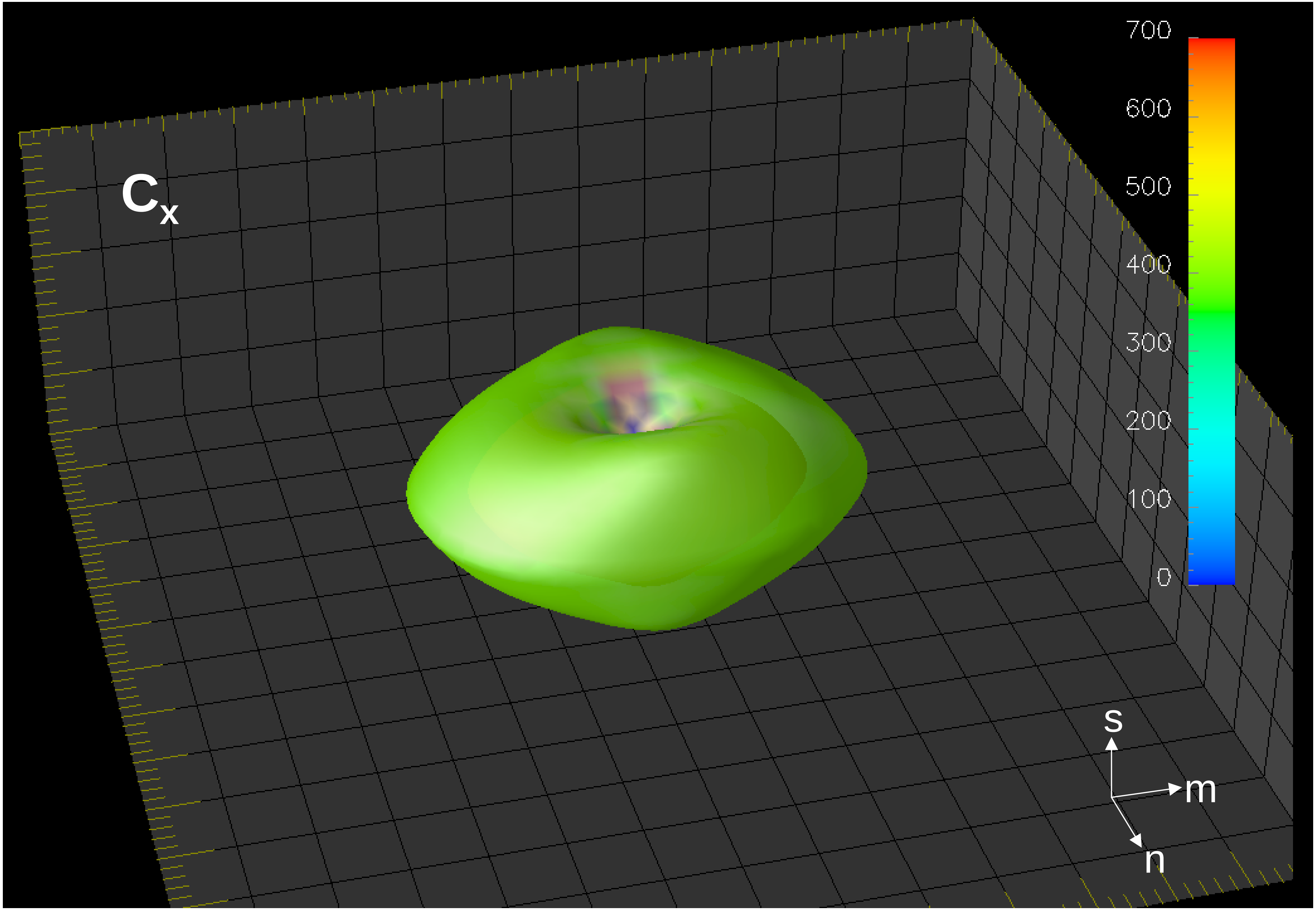
C$_x$
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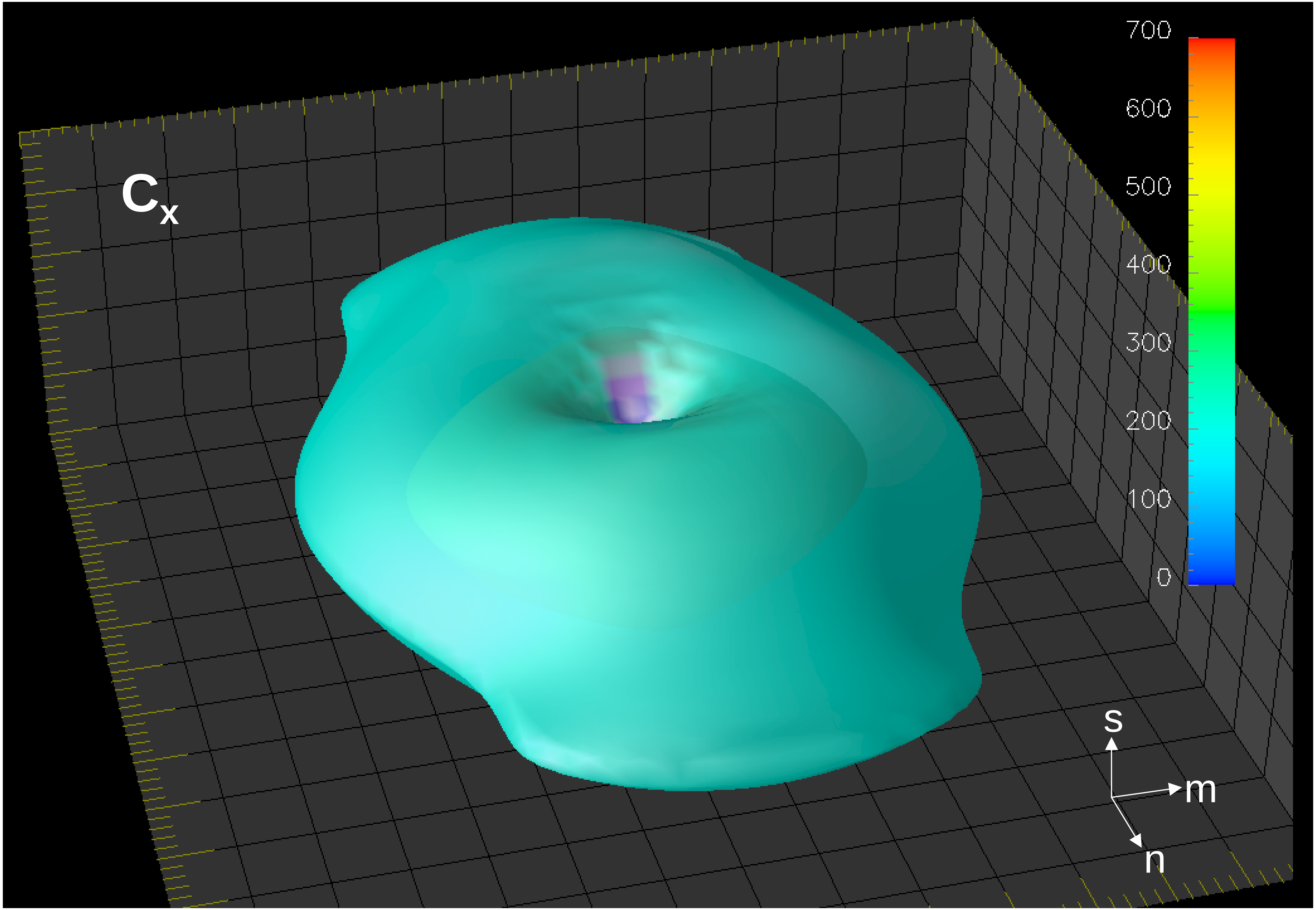

C_x
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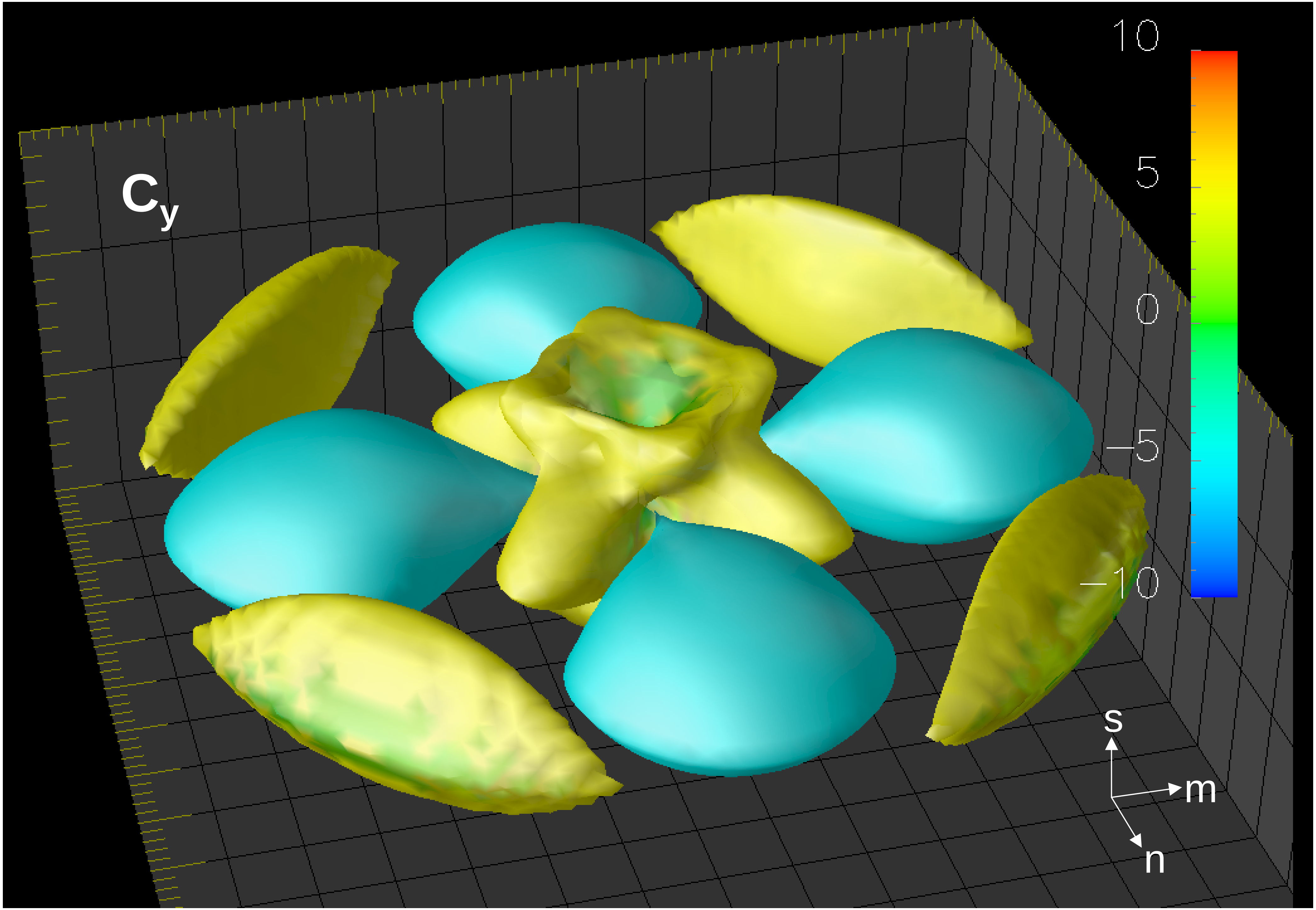

C_y
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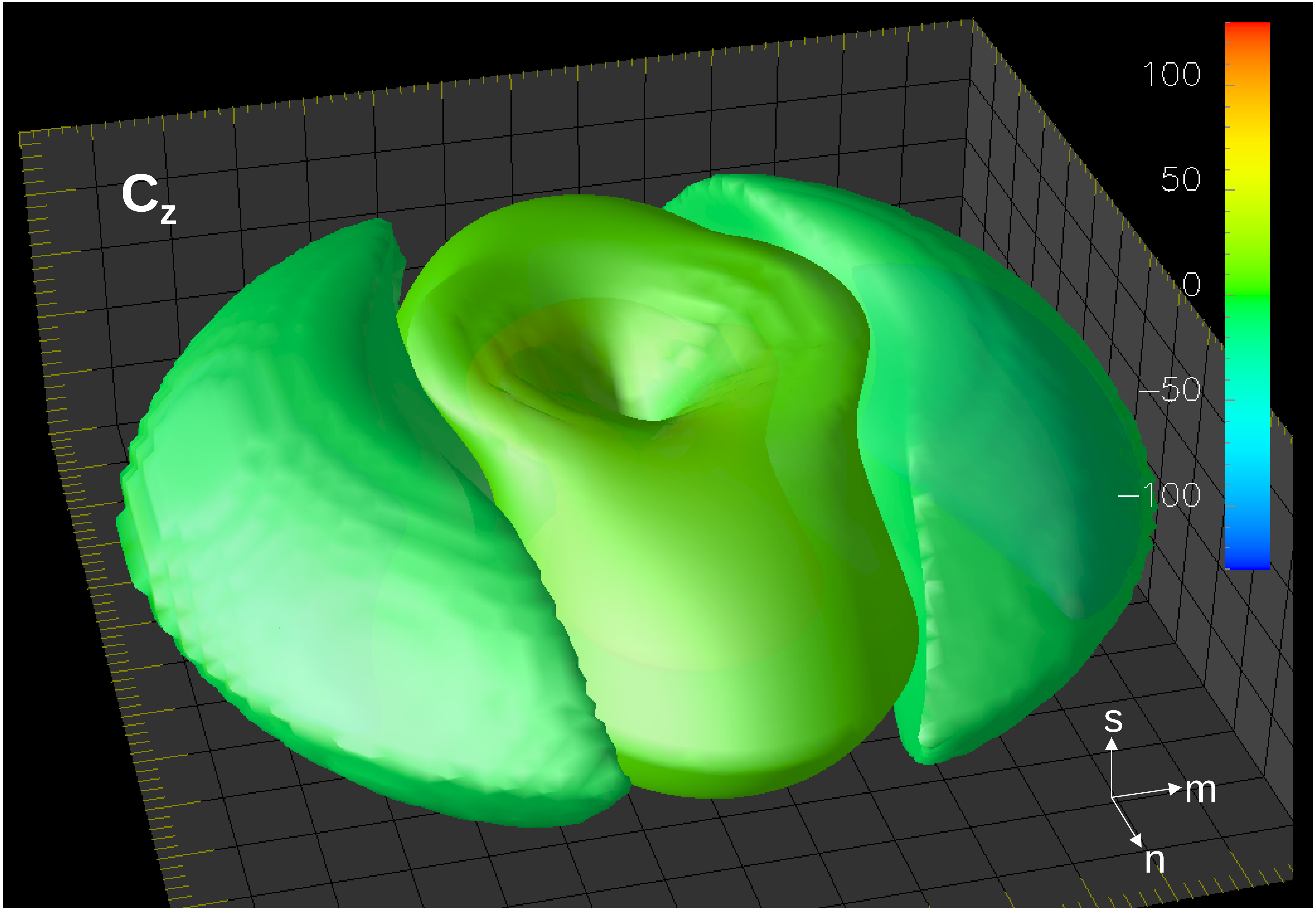

C_z
100
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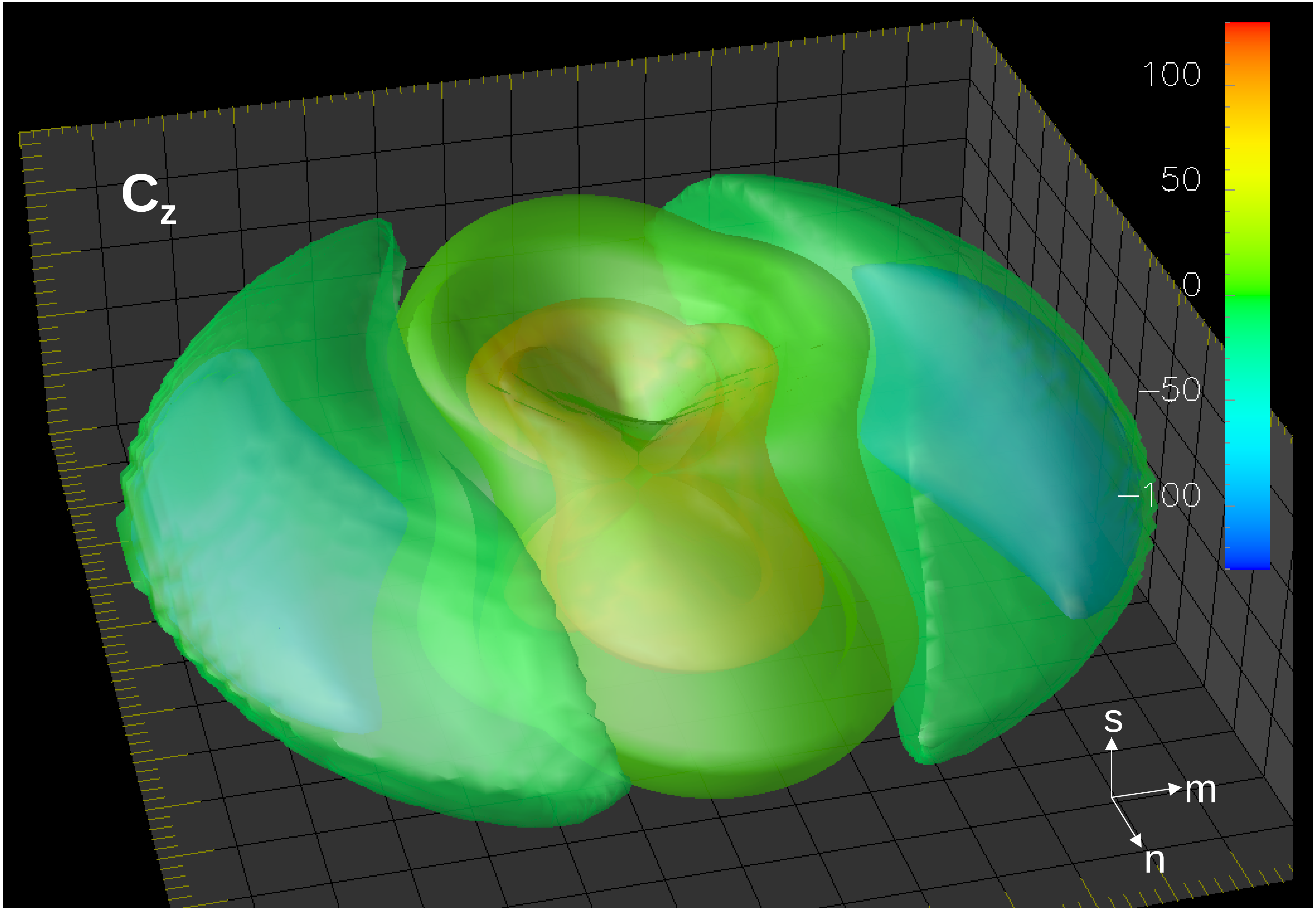

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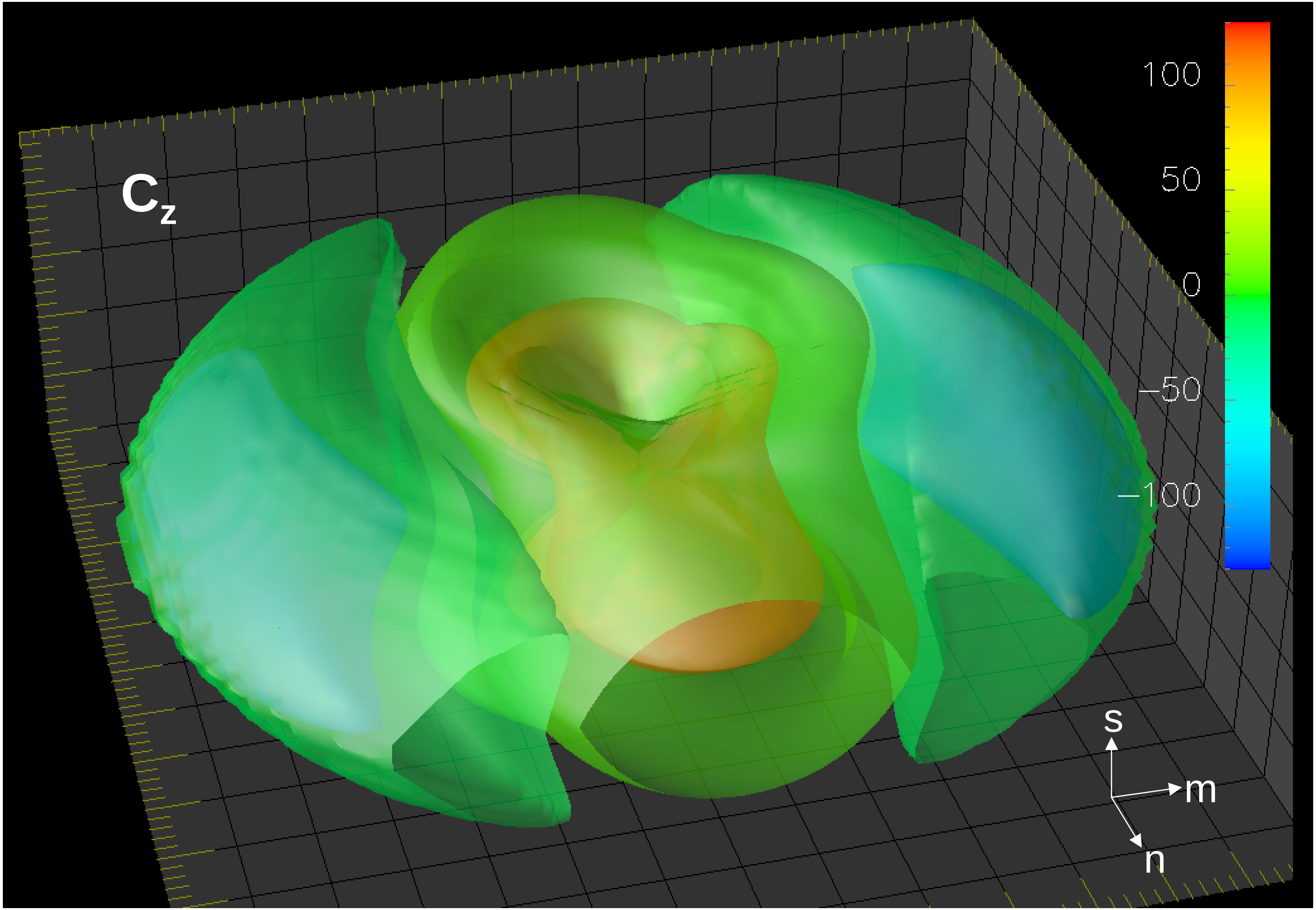
C_z
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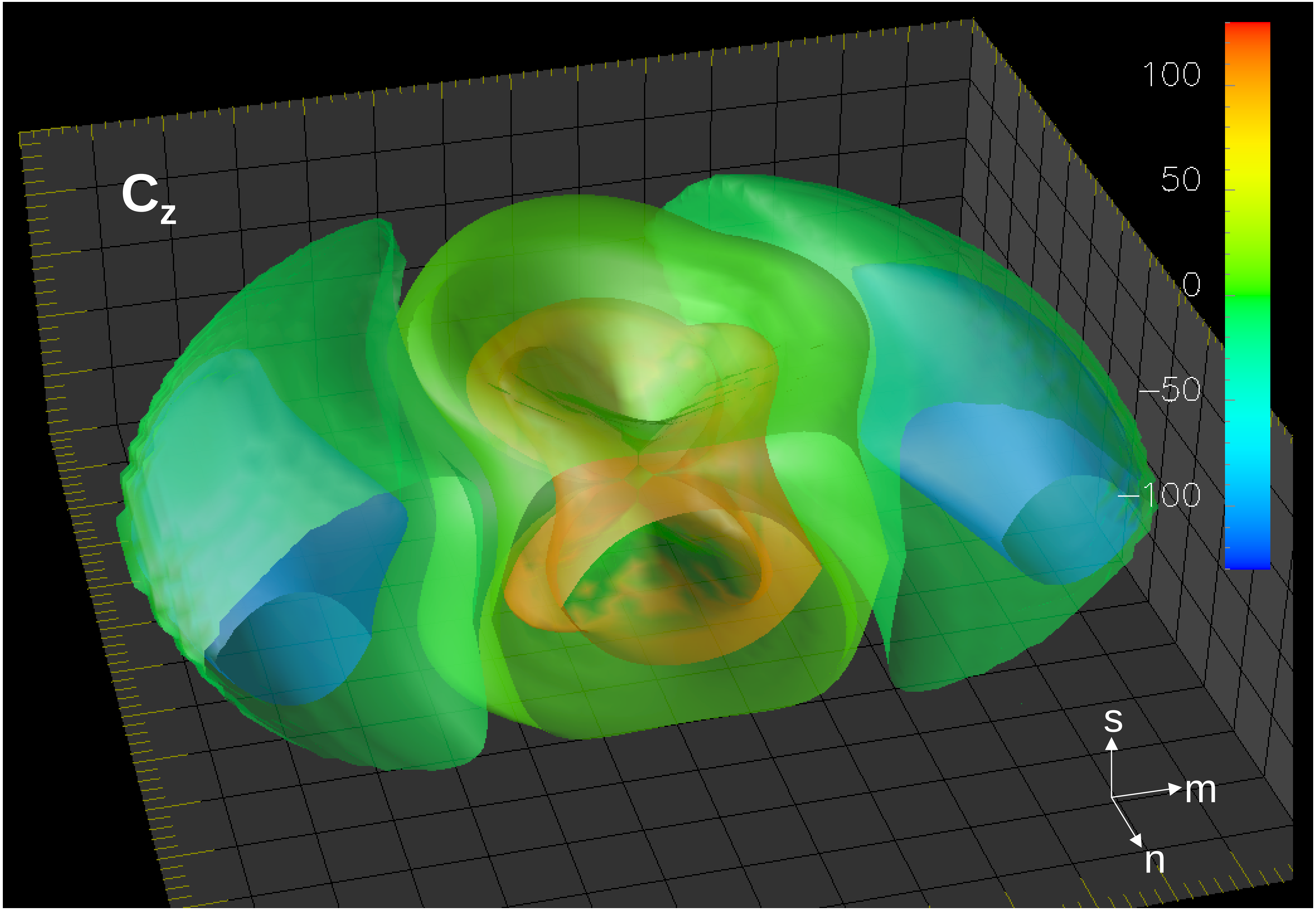

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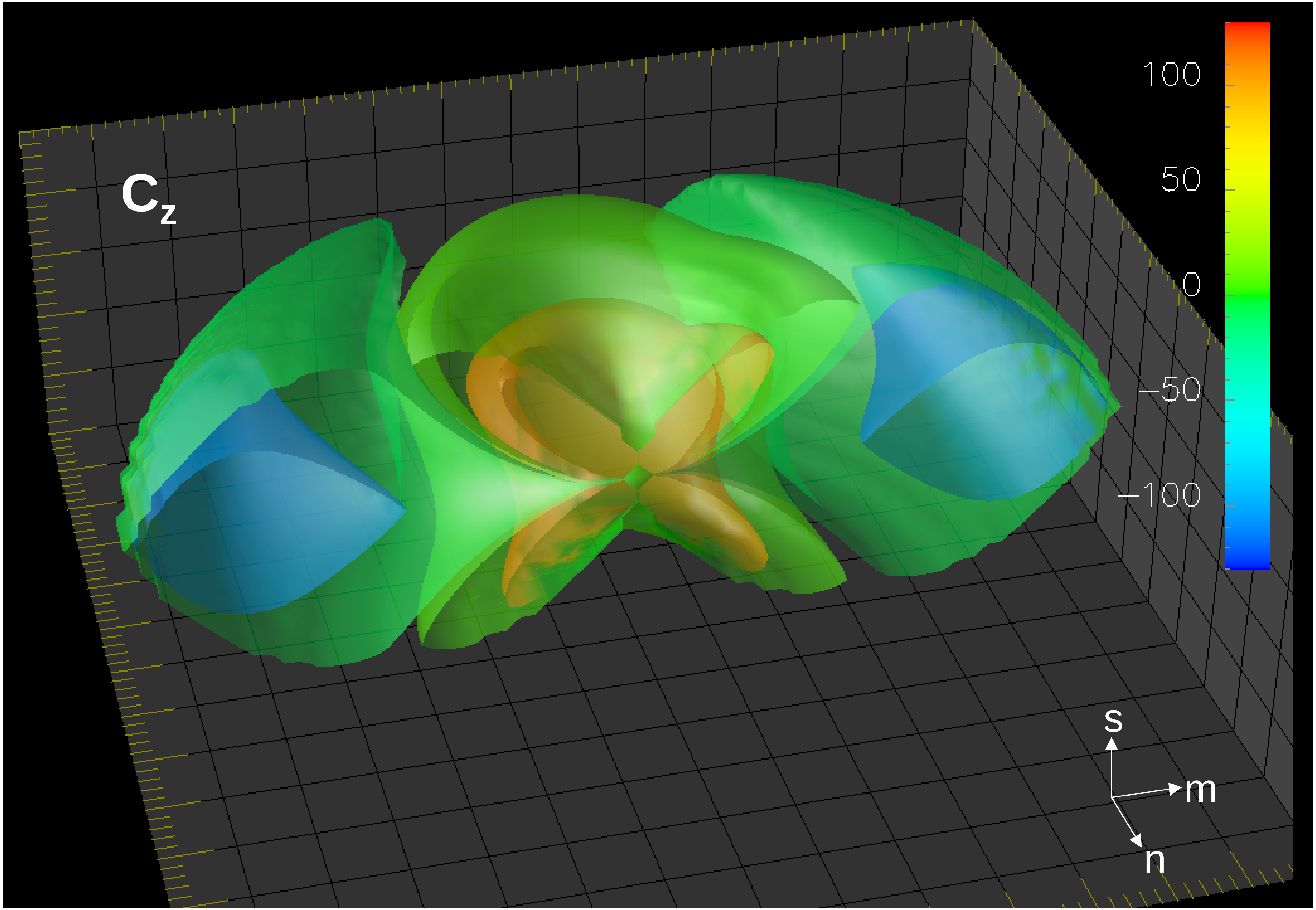

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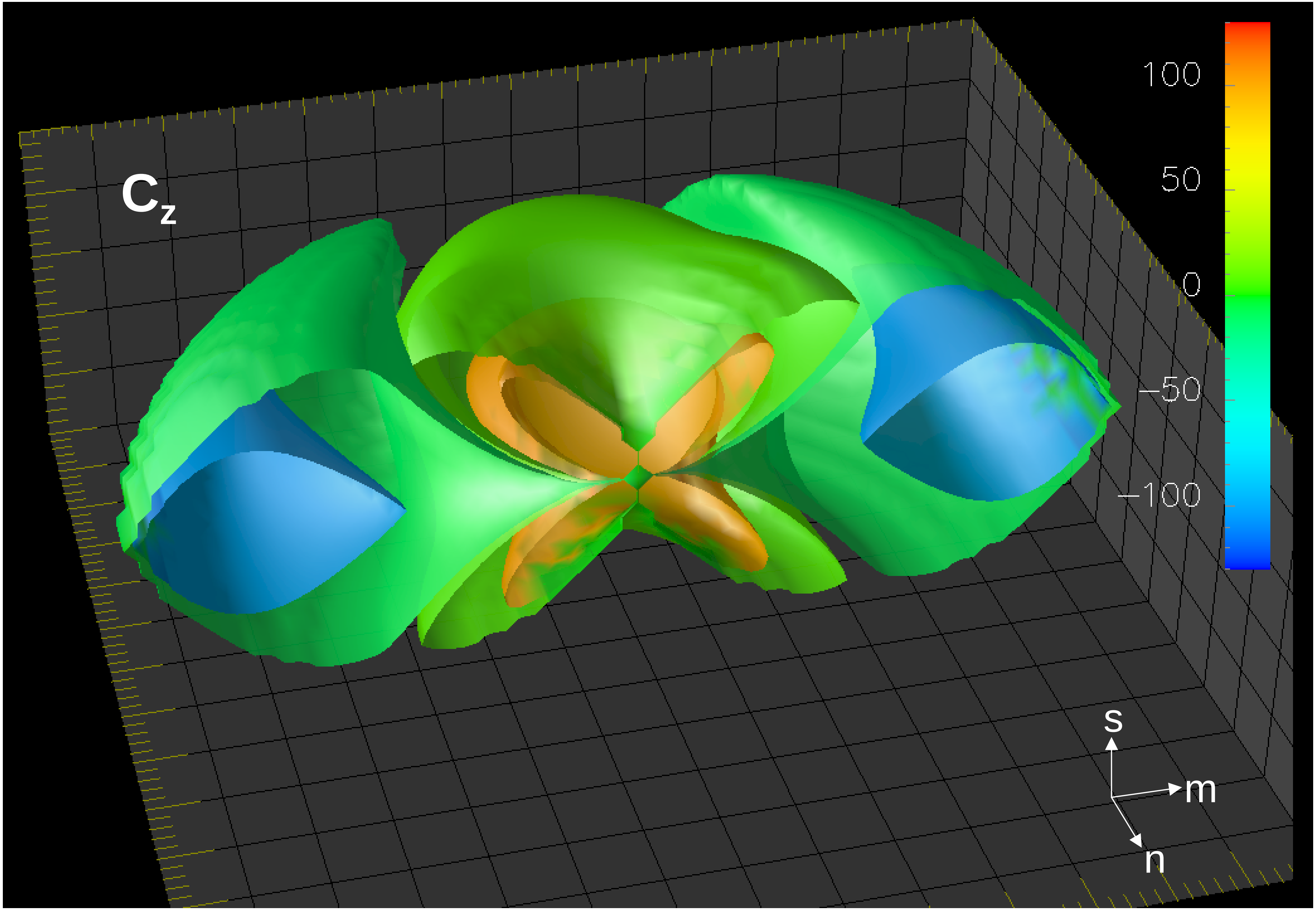

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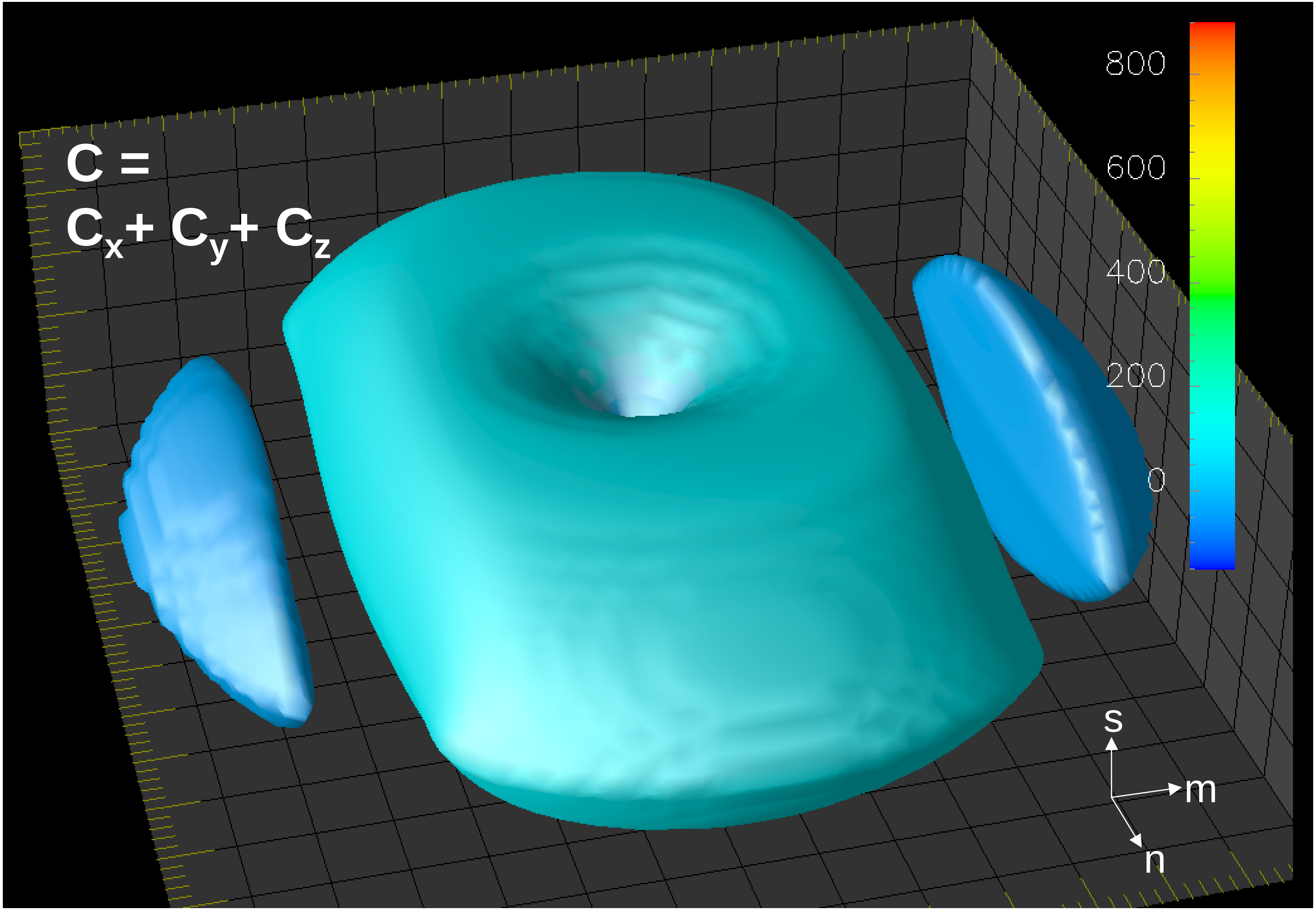

C =
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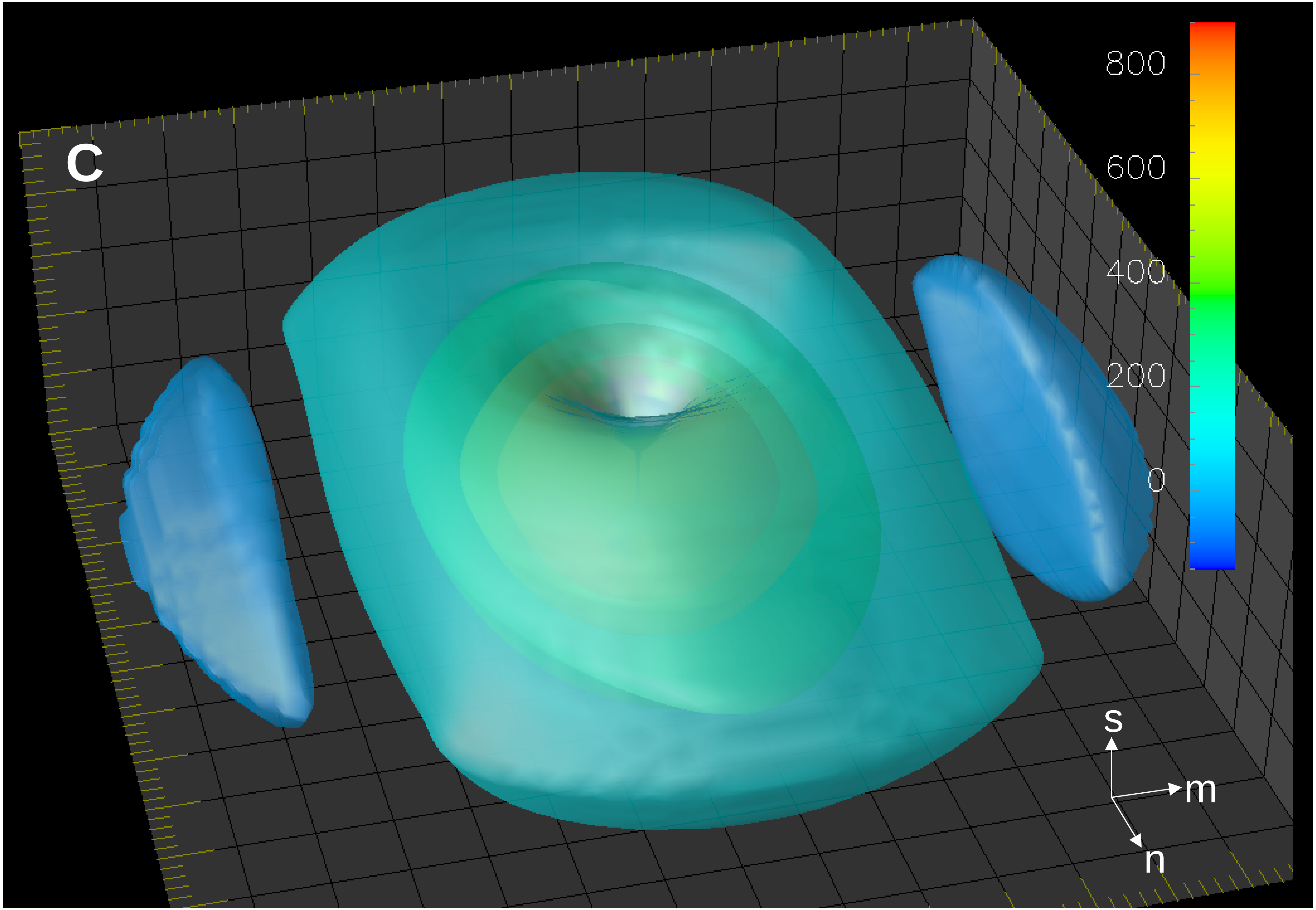
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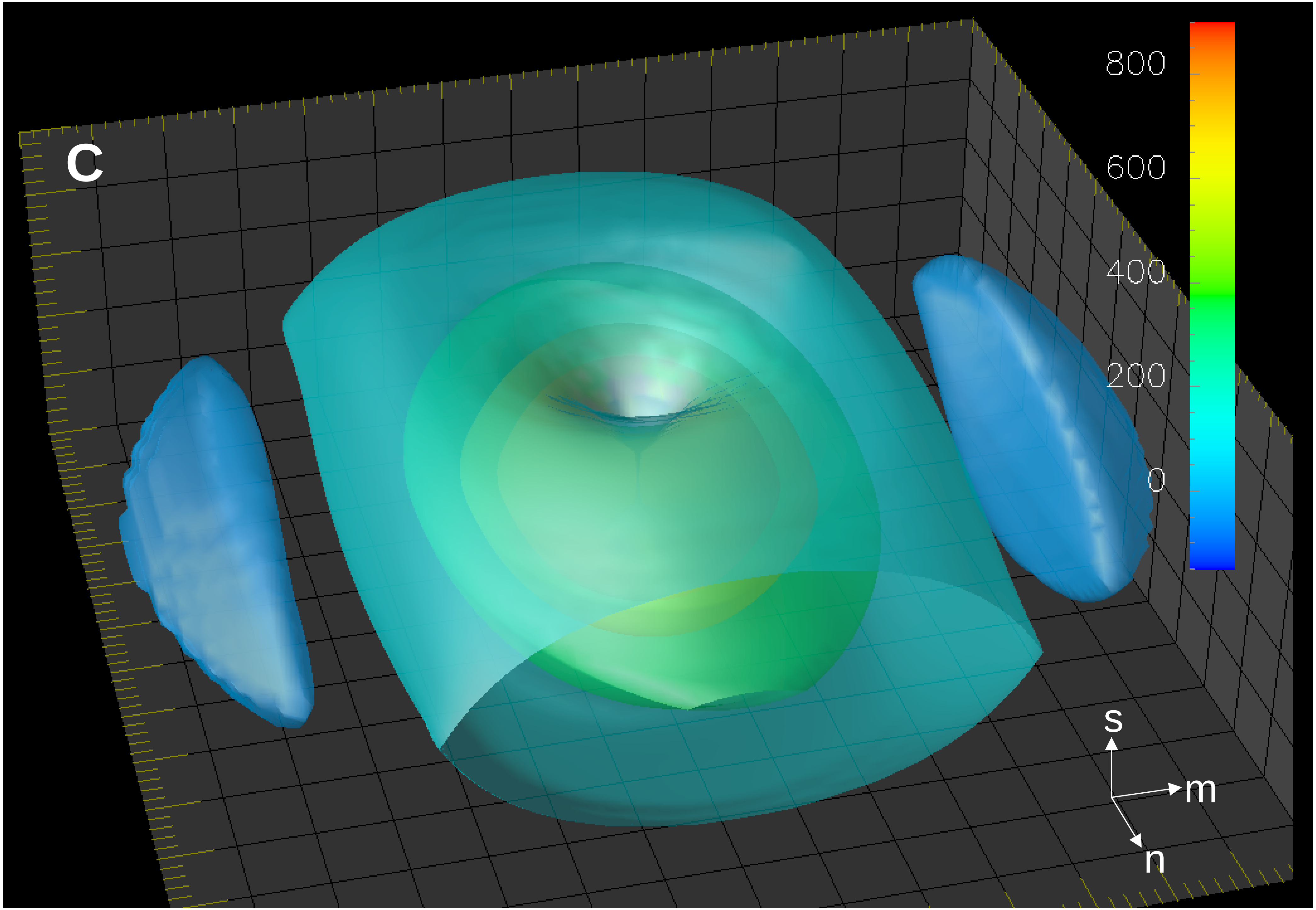
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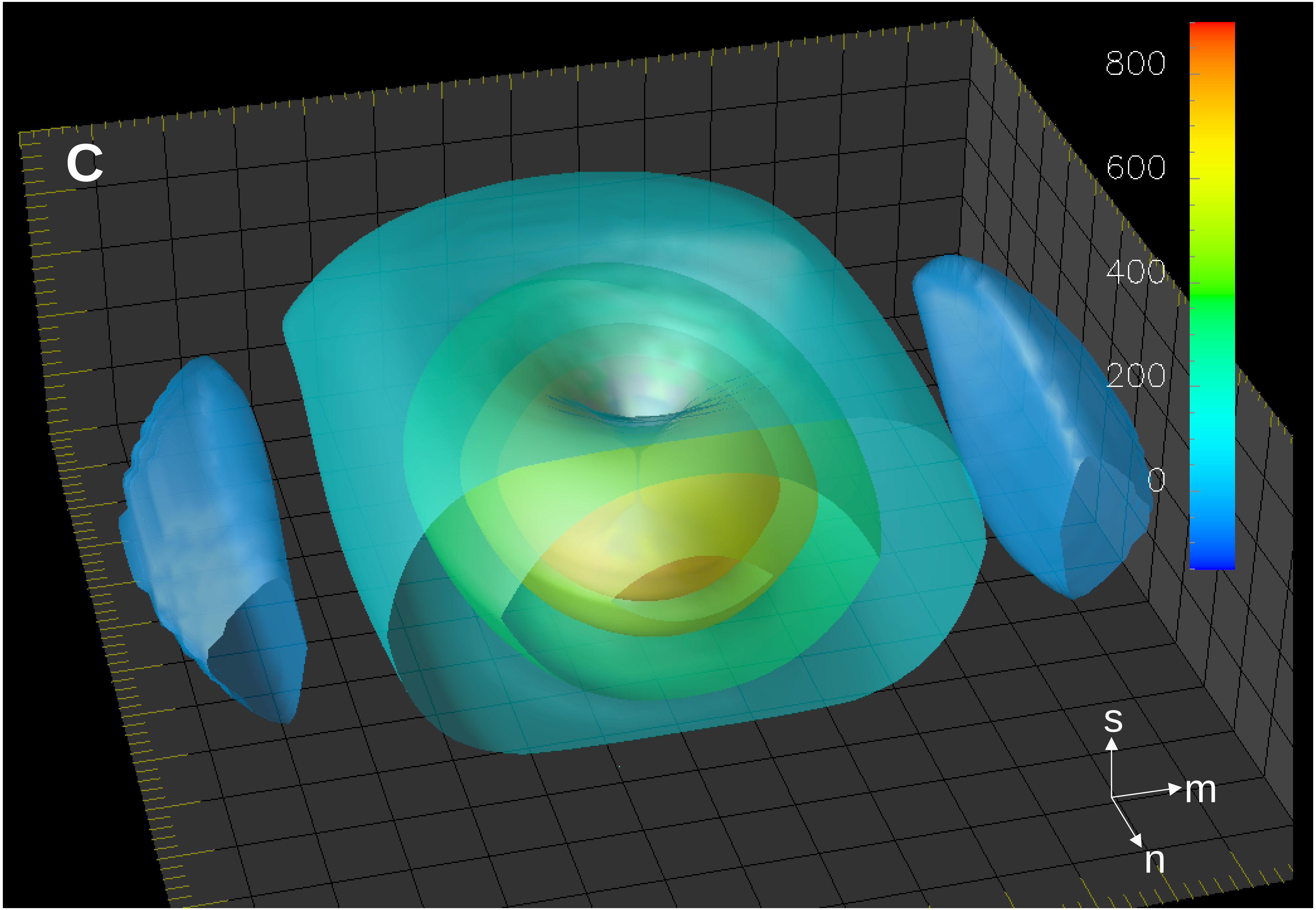
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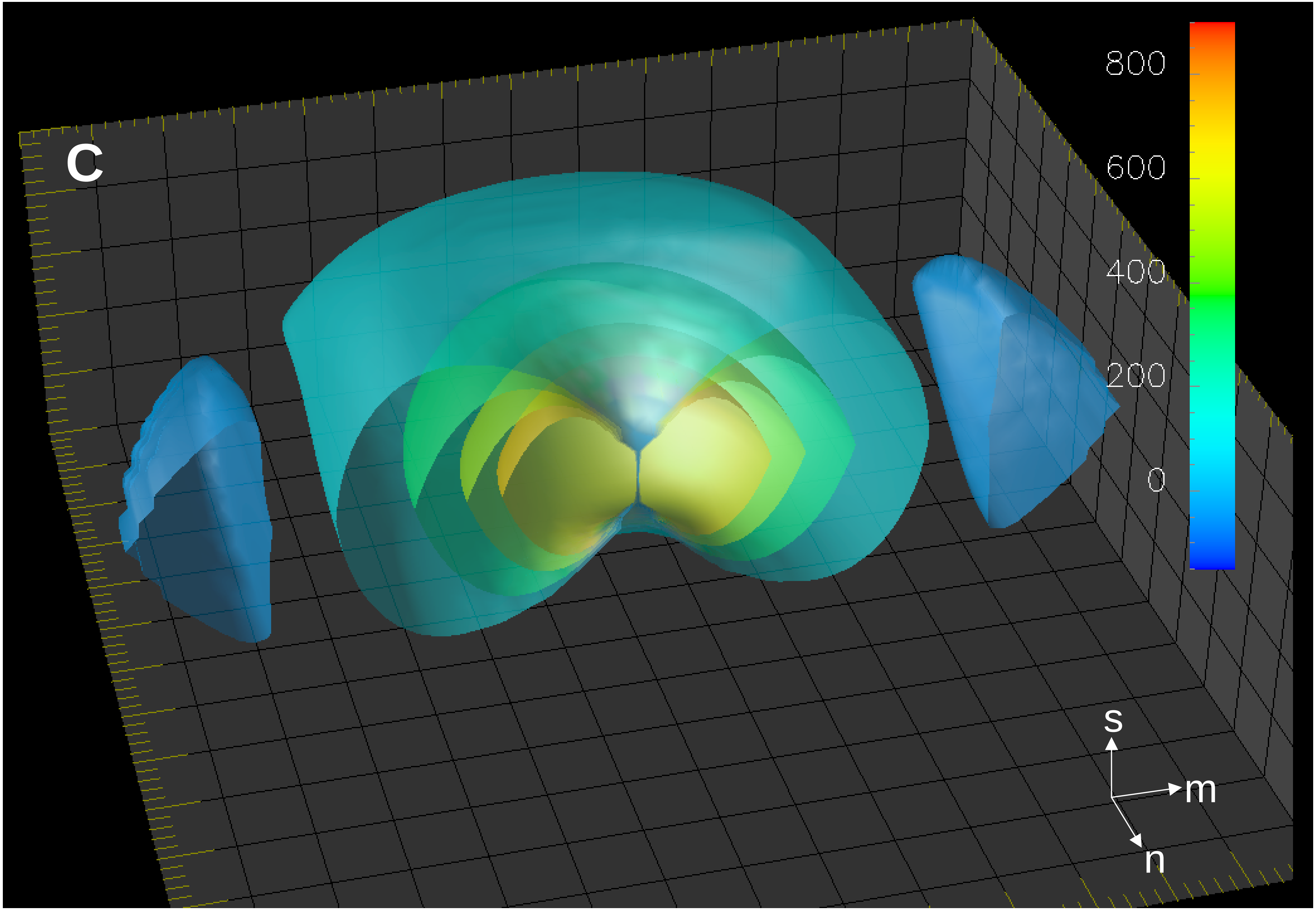
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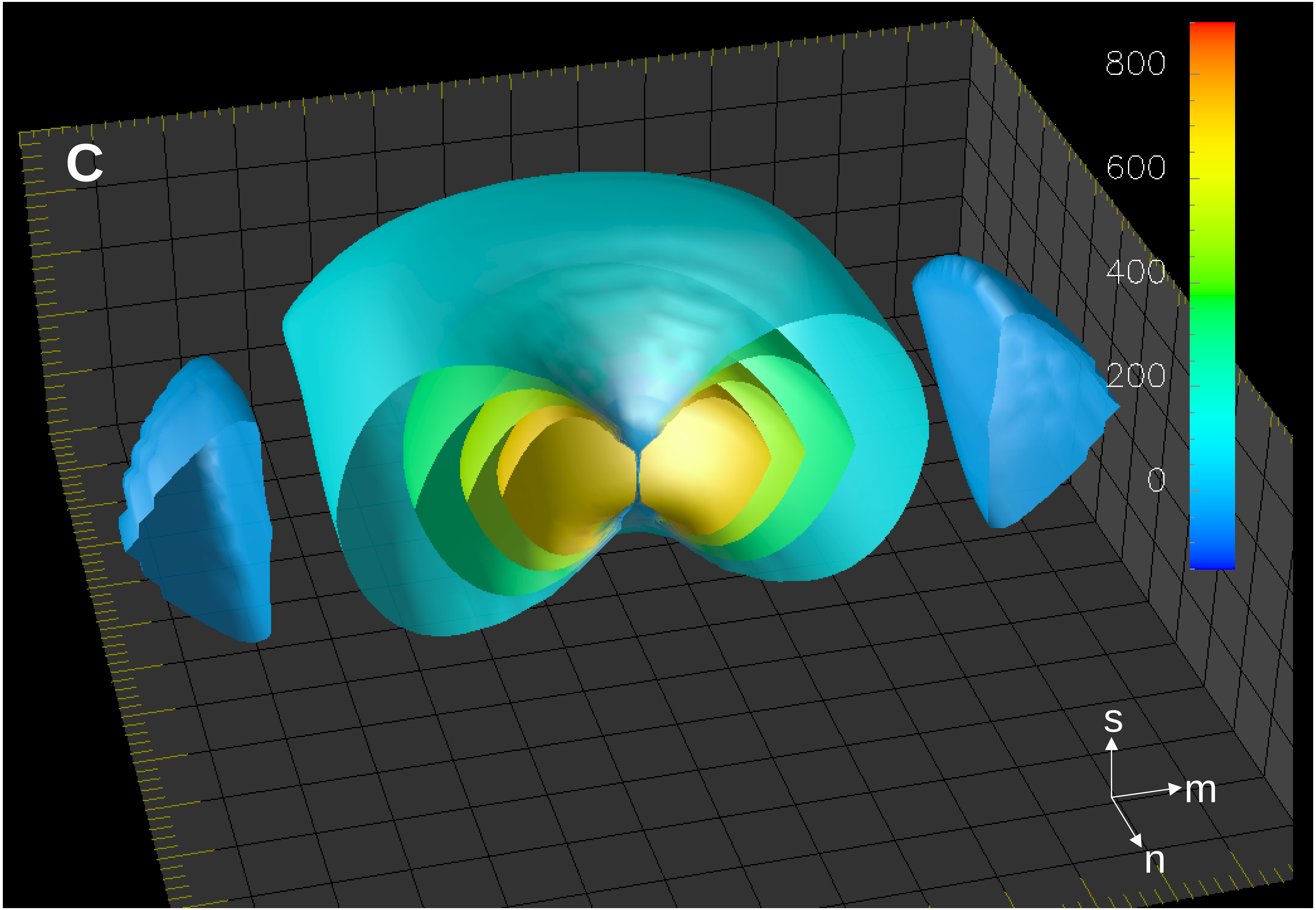
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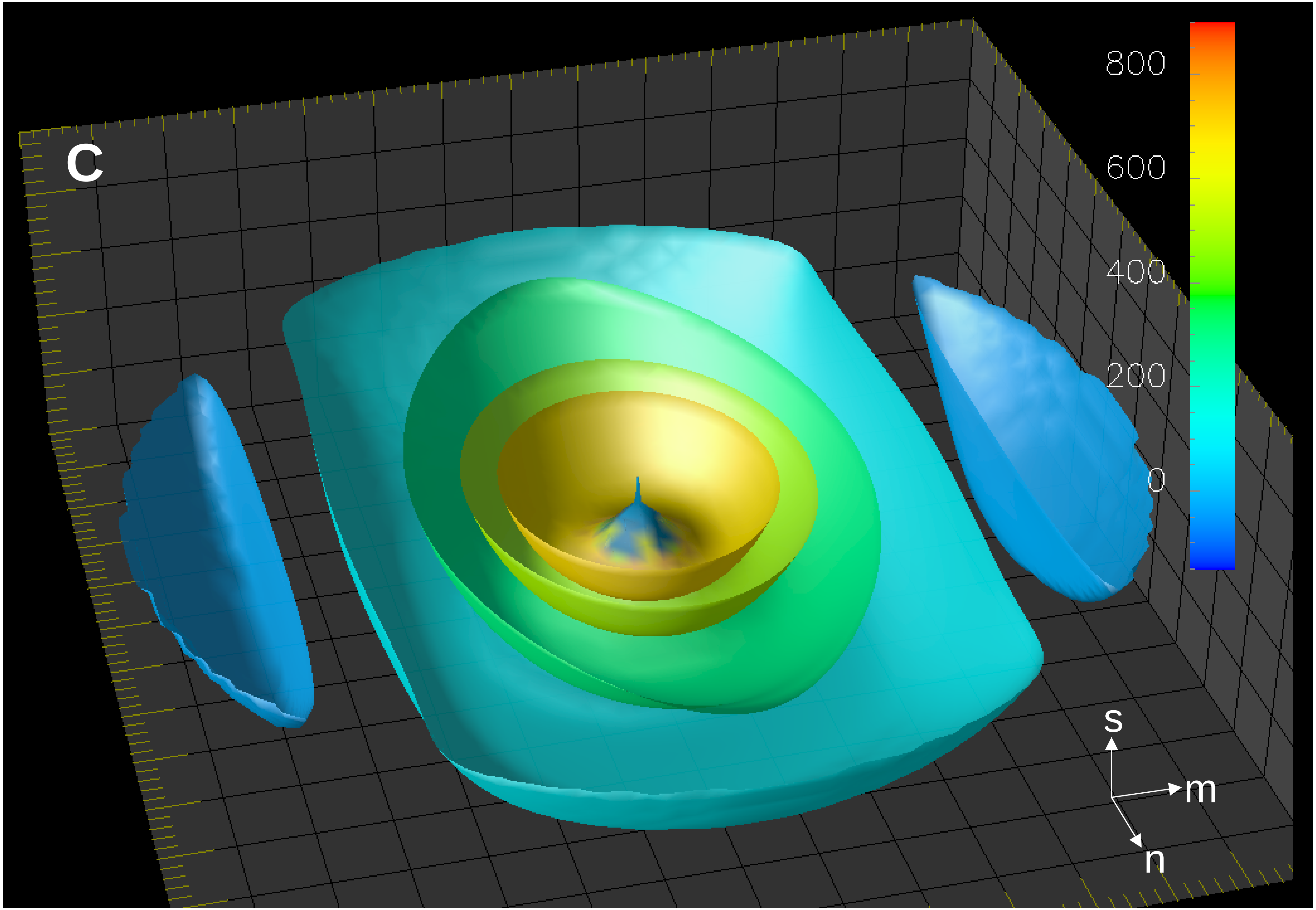
C
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What does it all mean?

- The vectorial OTF is simply the frequency spectrum of the vectorial intensity point spread function: perhaps “transfer function” is a misnomer
- For weak scattering or fluorescence imaging, if the object response is insensitive to polarisation, you could use this for accurate modeling
- Might also be useful for analysing high NA polarisation microscopy
- Another way of exploring the symmetries of vectorial focusing

Conclusion

- We have developed a general vectorial 3D OTF for arbitrary pupil functions.

- It's straightforward to calculate for any point in frequency space - no massive FFT arrays required!

- So we have a Fourier version of vectorial focusing theory, suitable for high aperture analysis.

Stay tuned to *Optics Communications* for details.